Spectral element modeling of earthquake nucleation and spontaneous rupture on rate and state faults

Yoshihiro Kaneko[1], Nadia Lapusta[1,2], Jean Paul Ampuero[3]

1. Division of Geological & Planetary Sciences, Caltech, Pasadena, CA
2. Division of Engineering & Applied Science, Caltech, Pasadena, CA
3. Institute of Geophysics, Seismology & Geodynamics, ETH, Zürich
E-mail: ykaneko@gps.caltech.edu

1 Introduction

Nucleation and spontaneous dynamic propagation of earthquakes on rate and state faults have been successfully modeled in the framework of boundary integral methods (BIM) (*Rice and Ben-Zion*, 1996; *Lapusta et al.*, 2000). However, these studies have been mostly restricted to planar faults embedded into a uniform elastic space or half-space, due to the nature of BIM. At the same time, observations show complicated crustal structures (such as layering and fault damage zones) and non-planar fault geometries. It is important to include those factors into earthquake models, combining them with laboratory-derived constitutive fault relations such as rate and state friction. In this work, we use 3-D spectral element method (SEM) to model earthquake nucleation and propagation of spontaneous rupture on a vertical strike-slip fault governed by rate and state friction. Our ultimate goal is to develop a SEM framework for simulating long-term deformation histories, in terms of sequences of earthquakes and combination of seismic and aseismic sliding.

2 Spectral element method (SEM)

- Flexibility of a finite element method and accuracy of a pseudo-spectral method
- Successfully applied in computational fluid dynamics and seismic wave propagation
- Diagonal mass matrix and explicit time scheme

3 Boundary integral method (BIM)

- Only the boundary (on the fault) is discretized
- Wave propagation is accounted for analytically through theoretically derived kernels
- Accurate and efficient but relatively limited in terms of geometry and bulk properties

4 Rate and state dependent friction

In the standard formulation with constant effective normal stress $\bar{\sigma}$, the shear strength τ can be expressed as:

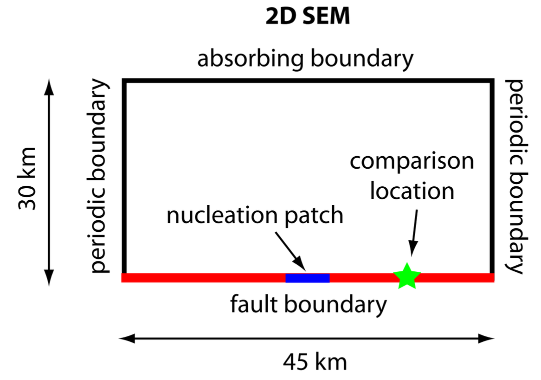
$$\tau = \bar{\sigma}\mu = \bar{\sigma}\left[\mu_0 + a\ln(V/V_0) + b\ln(V_0\theta/L)\right]$$

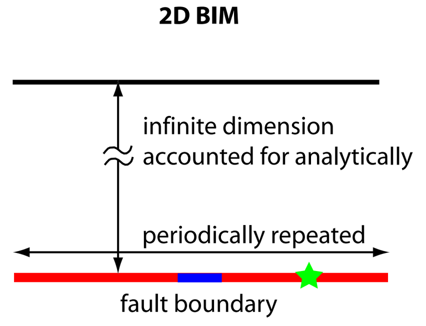
$$d\theta/dt = 1 - V\theta/L \quad \text{(aging law)}$$

$$d\theta/dt = (V\theta/L)\ln(V\theta/L) \quad \text{(slip law)}$$

- Formulated based on lab experiments
- Direct effect can be derived from a model of viscoelastic creep
- Capable of representing stable and unstable sliding
  - Potentially, unstable sliding when $a < b$
  - Stable sliding when $a > b$

5 Anti-plane test problem to compare SEM and BIM





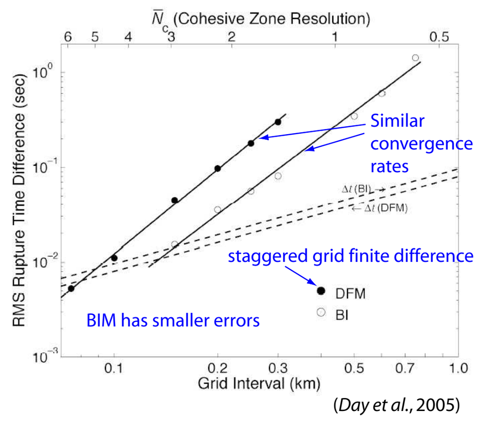
Our comparisons are similar to the SCEC code validation (*Harris et al.*, 2004), and we consider the following questions:

- the accuracy of SEM solutions with respect to BIM
- appropriate measures of errors
- abrupt vs. smooth initial conditions
- the state-variable updates, integration or direct use of evolution law
- simulations with aging law vs. slip law

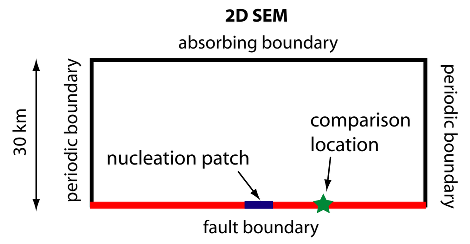
6 SCEC 3D code comparison

A study of Day et al., 2005, similar to our comparison, is based on:

- linear slip-weakening friction
- non-smooth initial conditions (both in time and space)
- the errors are represented as rupture arrival time (slip rates at 1 mm/sec)



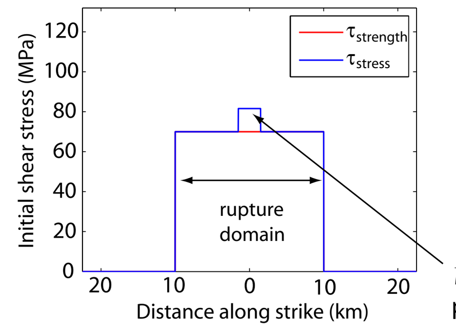
7 Initial conditions for our test cases



Rate and state friction

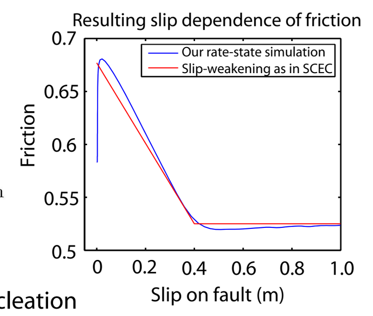
$$\tau = \bar{\sigma}\left[\mu_0 + a\ln(V/V_0) + b\ln(V_0\theta/L)\right]$$

$$d\theta/dt = 1 - V\theta/L \quad \text{(aging law)}$$



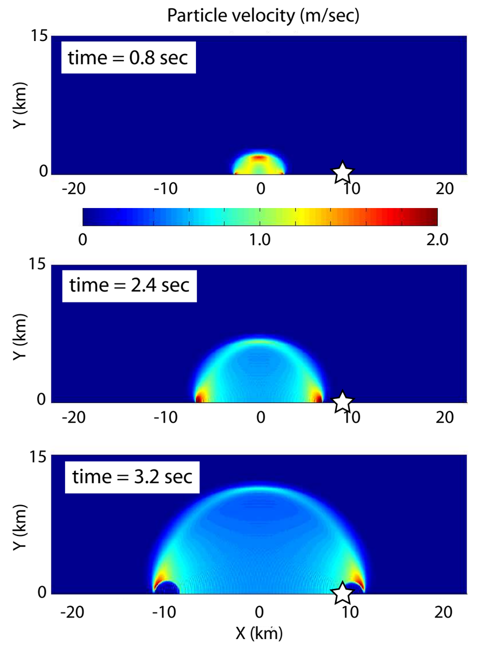
Within the rupture domain,
$\bar{\sigma}$ = 120 MPa
$V_{\text{initial}} = 10^{-3}$ m/sec
$\theta_{\text{initial}} \approx 10^2$ sec
$\tau_{\text{initial stress}} \neq \tau_{\text{initial strength}}$

V increases abruptly in the nucleation patch ("abrupt" initial condition)

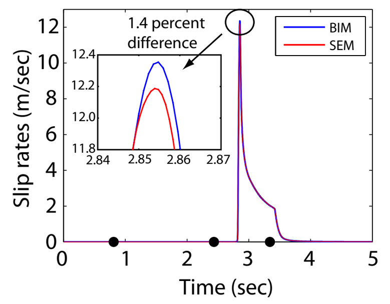


To have similar effective slip weakening of friction at the comparison location, we choose $a = 0.013$, $b = 0.018$, and $L = 0.038$ m. Hence, our initial conditions are similar to the initial conditions of the SCEC comparison problem.

8 2D SEM simulations using rate-state (aging law) friction



Our SEM "best run":
# of elements along X = 1215
# of elements along Y = 405
# of GLL nodes per element = 5
Average node spacing = 9.2 m
Δt = 0.89 msec

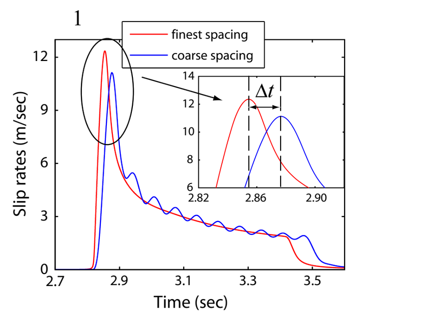


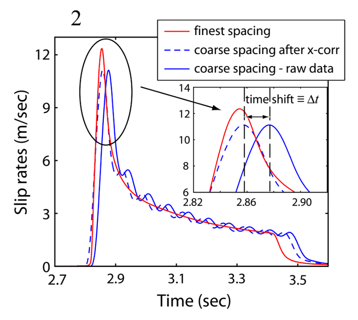
The rupture nucleates at the center of the fault, and SH wave propagates in the medium. The star indicates our comparison location. The right figure represents the SEM and BIM slip rates as a function of time at the comparison location for the best-resolution runs.

9 Measures of errors: rupture arrival time vs. cross-correlation

Two different measures of errors

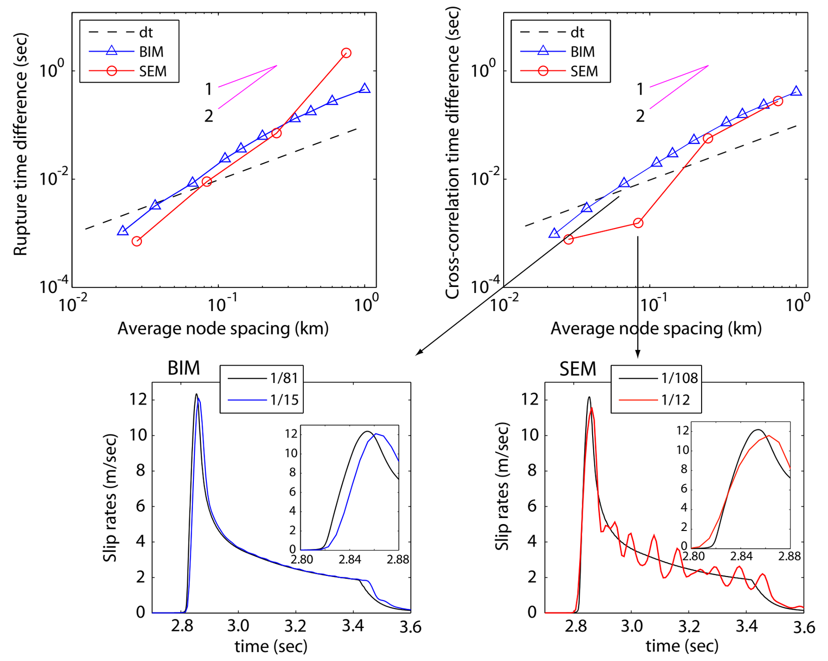
1. Rupture arrival time difference
   Rupture arrival time ≡ time at a peak of slip rates (our current definition)
2. Cross-correlation time difference



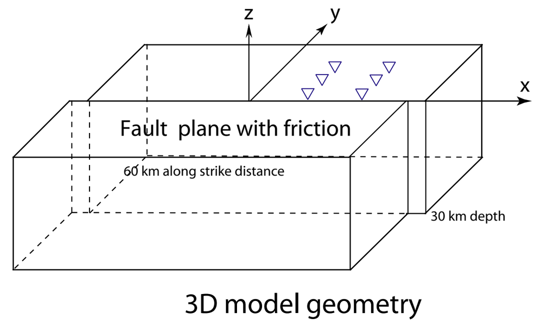


10 Convergence rates: rupture arrival time vs. cross-correlation

On the vertical axis, the rupture arrival time difference or the cross-correlation time difference between the highest-resolution run and lower-resolution runs is shown. In general, BIM and SEM give similar convergence rates in terms of the rupture arrival time difference. When the cross-correlation time difference is used, SEM shows smaller errors than those of BIM. For one of such cases, the slip rates for both BIM and SEM are shown on the bottom panels.

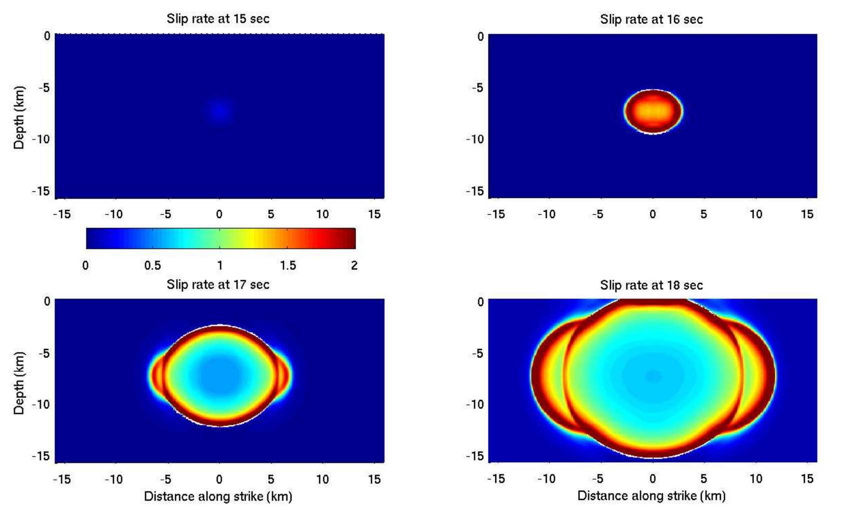


11 Geometry of 3D simulation



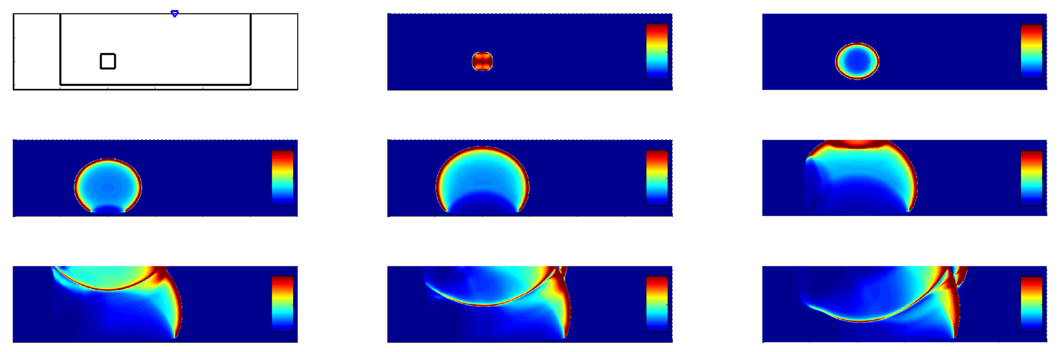
3D model geometry

12 Nucleation and super-shear transition of spontaneous rupture



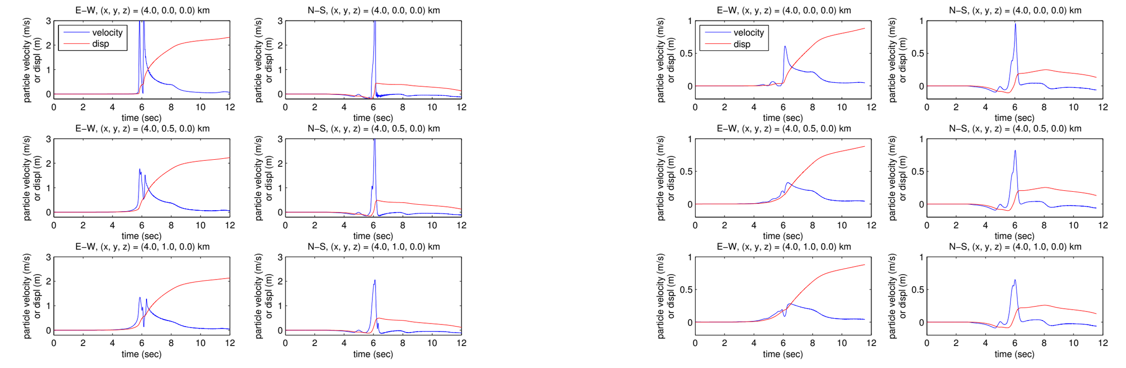
We have incorporated rate and state friction into 3D SEM dynamic rupture code (*Ampuero*, 2004). The snapshots of the slip rates on the fault are shown above. The initial conditions used for this simulation are similar to the smooth case in 2D, where the nucleation proceeds gradually. The rupture speed transitions from sub-shear to super-shear in the in-plane direction, consistently with daughter crack mechanism of Burridge-Andrews for slip weakening friction (Andrews, 1976). Note that the transition is observed here for rate and state friction laws.

13 Dynamic rupture simulation in homogeneous media

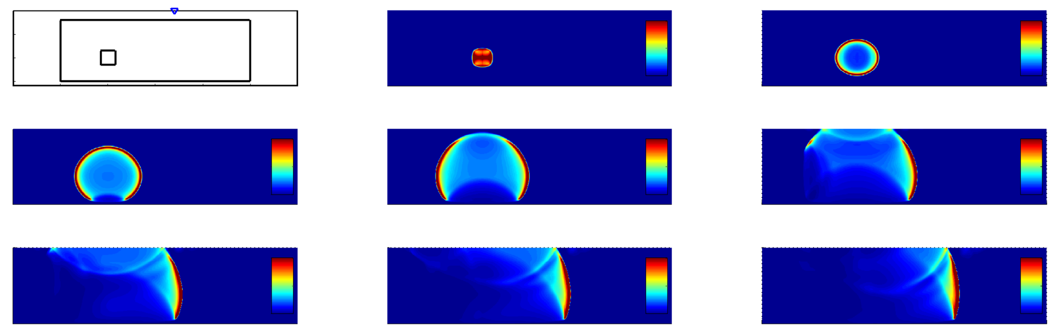


14 Dynamic rupture simulation in homogeneous media (continued)

Snapshots of horizontal slip rate (m/s) on the fault in the interval of one second. Homogeneous property (Vs = 3464 m/s, Vp = 6000 m/s) is imposed. The quantity $(a-b) = -0.005$ in the vel-weakening region, 0.005 in the vel-strengthening region. Effective normal stress is 120 MPa, and initial shear stress is 70 MPa except at "nucleation region" (81.6 MPa). The rupture speed becomes super-shear near the free surface.

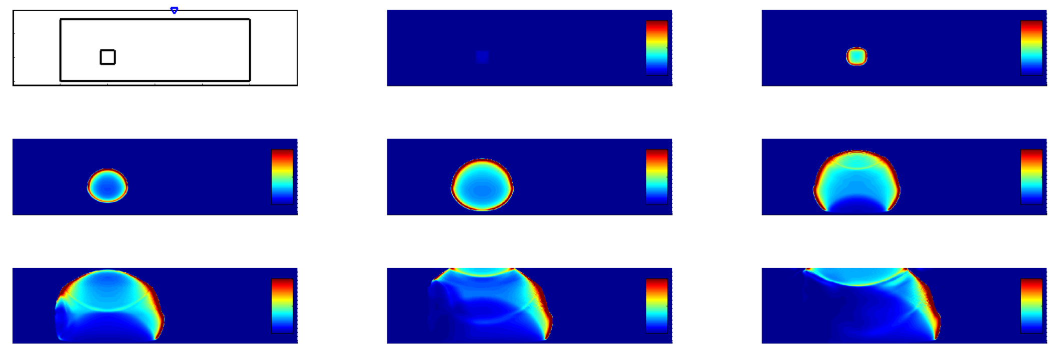


Displacement and velocity seismograms at 4-km horizontal distance (14 km away from the center of nucleation). Right and left insets correspond to the simulations above and below respectively. The top, middle, and bottom panels correspond to different off-fault locations (on the fault, 0.5 km away, and 1.0 km away from the fault).



Same as above except that the effective normal stress gradually increases with depth and constant (120 MPa) below 6-km depth. The transition from velocity-weakening to velocity-strengthening is smooth and occurred at 2-km depth.

15 Dynamic rupture simulation in a layered structure



Snapshots of horizontal slip rate (m/sec) on the fault are shown in the interval of one second. Layered velocity model is used (Vs = 2136 m/s and Vp = 3700 m/s for 0 - 4 km depth, Vs = 2887 m/s and Vp = 5000 m/s for 4 km - 10 km depth, Vs = 3464 m/s Vp = 6000 m/s for 10-30 km depth).

16 Conclusions

- We have incorporated rate and state friction into 2D and 3D SEM dynamic rupture code for simulating a single earthquake.
- SEM and BIM give virtually indistinguishable solutions to the test problem with the nucleation and spontaneous rupture propagation when the node spacing is small enough.

17 Future work

- Understand how much can be learned from the near-field seismic records in terms of the history of slip or slip rate on the fault with different weakening mechanisms.
- Develop SEM to include variable time steps to simulate long-term deformation history of a fault.