An Instantaneous Sub-Rayleigh-to-Supershear Transition Mechanism

Yi Liu[1] and Nadia Lapusta[1,2]

1. Division of Engineering & Applied Science, Caltech, Pasadena, CA
2. Division of Geological & Planetary Sciences, Caltech, Pasadena, CA
E-mail: yil@caltech.edu, lapusta@caltech.edu

1 Abstract

Observations suggest that supershear bursts and sustained supershear propagation may have occurred in a number of earthquakes (such as 1992 Landers, 1999 Izmit, and 2001 Kunlun). Two mechanisms have been proposed to explain the sub-Rayleigh-to-supershear rupture transition. In the Burridge-Andrews mechanism (e.g., Andrews, 1976), the shear-wave stress peak traveling in front of the main Mode II rupture nucleates a daughter crack, if the prestress on the fault is large enough, and the daughter crack is born supershear. Alternatively, supershear burst can result from breaking a strong heterogeneity on a 3D fault as shown by Dunham et al. (2003).

Taking a broader look at the Burridge-Andrews mechanism, we hypothesize that, for the model to produce rupture transition from sub-Rayleigh to supershear speeds, it needs to accomplish two goals: (i) nucleate a crack and (ii) drive the crack fast enough. These processes are inseparable in the Burridge-Andrews mechanism, as they both occur at the shear stress peak of the main crack propagating in a uniform prestress field. Our simulations show that goals (i) and (ii) can be achieved in other ways. For example, one can advance Mode II rupture towards a location susceptible to crack nucleation, such as a preexisting subcritical crack, a patch of lower static friction strength, or a patch of higher prestress. In these cases and under the right conditions, the secondary crack nucleates before the shear stress peak arrives, and yet the stress field of the advancing main rupture is still able to drive the secondary crack to supershear speeds. Hence nucleating the daughter crack at the shear stress peak, as it is done in the Burridge-Andrews mechanism, is not essential for the subsequent supershear propagation. One can also use different means of driving the crack supershear, such as overstressing statically a part of the rupture or imposing an outside dynamic stress field.

We observe the following interesting features in our simulations, which we will present along with our preliminary analysis:

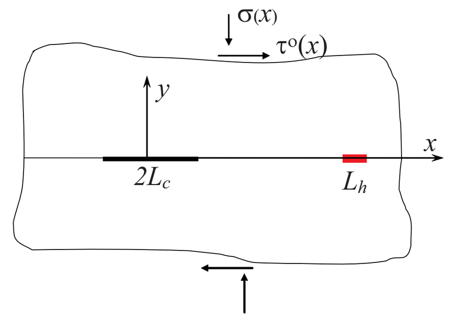
(1) Crack fronts can abruptly jump from the Rayleigh-wave speed to a supershear speed. We call this "direct" supershear transition. For example, consider a secondary crack nucleated by one of the ways described above under the advancing stress field of the main rupture. The secondary crack is sub-Rayleigh and it accelerates towards the Rayleigh wave speed. Once the Rayleigh wave speed is reached, the secondary crack jumps to a supershear speed instantaneously.

(2) The supershear transition mechanisms we have described work not only in 2D in-plane models, but also in 3D models under certain conditions.

(3) Once the transition takes place in our models, the supershear rupture propagation can be maintained under prestress levels that are much lower than the ones predicted by the Burridge-Andrews mechanism. This shows that the level of prestress implied by the Burridge-Andrews mechanism is only needed to nucleate a crack at the site of the shear-wave peak, and not to drive the rupture to supershear speeds or to maintain that supershear propagation.

2 Simulated Model (2D)

A planar interface is embedded in an infinite, elastic and homogeneous space. The main rupture initiates from a length of $2L^c$ given in (2). In some cases, a heterogeneity exists in front of the main rupture. Depending on simulated problems, the heterogeneity may be a preexisting subcritical crack, a patch with lower static friction strength, or a patch with higher prestress.



The fault strength Γ is assumed to be governed by a linear slip-weakening law:

$$\Gamma(\delta) = \begin{cases} \tau^d + (\tau^s - \tau^d)(1 - \delta/d_o) & \delta \le d_o \\ \tau^d & \delta > d_o \end{cases} \quad (1)$$

A singular shear crack with uniform prestress τ^o will propagate spontaneously if its half length exceeds a critical value L^c (*Andrews 1976*):

$$L^c = \frac{2}{\pi}\frac{\mu(\lambda+\mu)}{(\lambda+2\mu)}\frac{(\tau^s-\tau^d)d_o}{(\tau^o-\tau^d)^2} \quad (2)$$

The seismic ratio S (*Andrews 1976*) quantifies the level of prestress and is defined as $S = (\tau^s - \tau^o)/(\tau^o - \tau^d)$. Without loss of generality, we assume $\tau_d = 0$ in simulations. Rupture propagation is calculated with grid size Δx and time step Δt:

$$\Delta x = \frac{L^c}{N^c} \qquad \Delta t = \frac{\Delta x}{\beta c_s} \quad (3)$$

3 Burridge-Andrews Supershear Transition Mechanism

Based on a self-similar shear crack model, Burridge (1973) found that there is a shear stress peak $\tau^l = \tau^o + S_{crit}\left(\tau^o - \tau^d\right)$, which propagates with the shear wave speed in front of the crack. In numerical simulations of a 2D in-plane shear crack governed by linear slip-weakening law, Andrews (1976) observed that the stress τ^m at the shear wave peak gradually increases as the rupture propagates, and approaches the limiting value τ^l. If $\tau^l > \tau^s$ (or equivalently, the seismic ratio $S < S_{crit} \approx 1.63$), τ^m overcomes the fault strength τ^s in the process of rupture propagation, and a daughter crack nucleates at the shear wave front, propagating with a supershear speed.

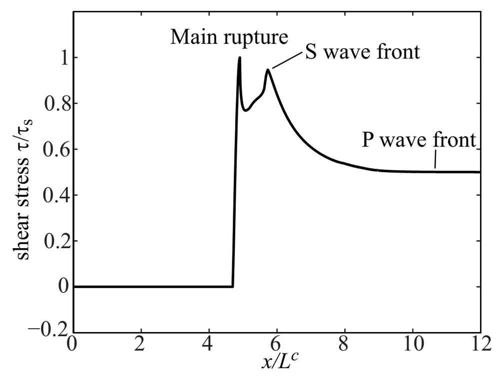


Figure: Shear stress distribution on the fault in Burridge-Andrews model.

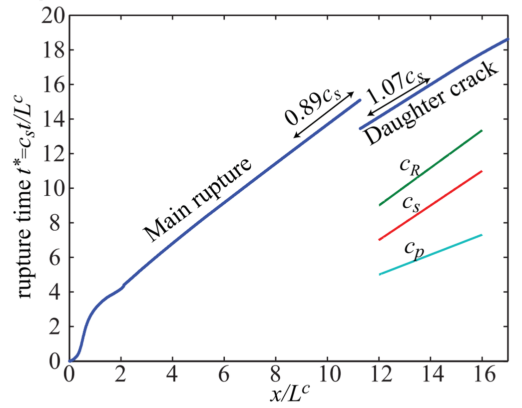


Figure: Rupture time along the fault in Burridge-Andrews model.

4 Supershear transition in our model

In the following, we will show that various other approaches, described in the abstract, are also able to trigger the rupture to go supershear.

4.1 Advancing main rupture toward a preexisting subcritical crack

To smoothly initiate main rupture and a subcritical secondary crack, we impose the following loading stress $\tau^o(x,t')$:

$$\tau^o(x,t') = \tau^o + (\tau^s - \tau^o)[1 + (1 - e^{-1})t'][e^{-\frac{x^2}{L_0^2}} + e^{-\frac{4(x-D)^2}{L_0^2}}] \quad t' < t_{crit} \quad (4)$$

where $L_o = 0.6\mu d_o/(1-\nu)\tau^s$ is half of the critical nucleation length for in-plane crack obtained by *Uenishi and Rice (2003)*, $D = 12L^c$, and $\tau^o = \tau^s/3$. This form of loading stress initiates two separate cracks at $x = 0$ and $x = D$ at $t' = 0$. At $t' = t_{crit}$, the length of the crack around $x = 0$ reaches the critical length $2L_o$, and it starts to propagate spontaneously; and the length of the crack around $x = D$ is only $1.2L_o$, therefore it remains a subcritical crack. We stop increasing the loading stress at $t' = t_{crit}$, and set $t = t' - t_{crit}$.

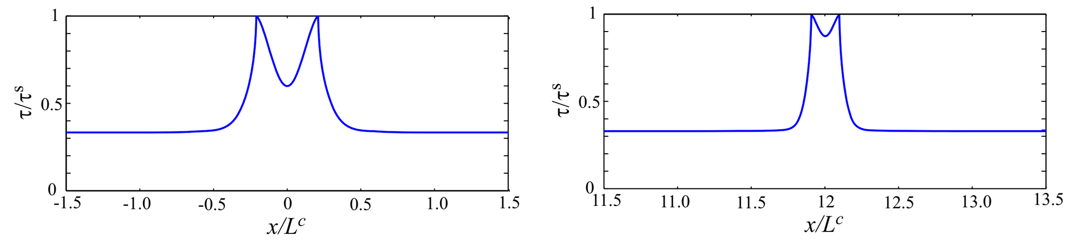
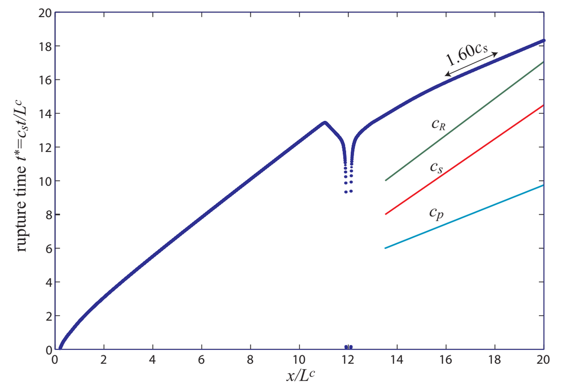
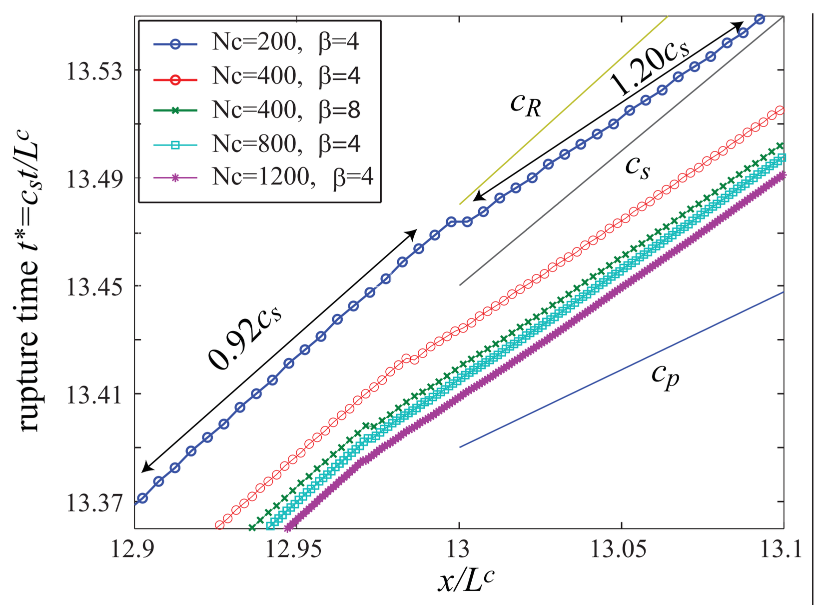


Figure: Stress distribution around the main and preexisting crack at time $t = 0$.

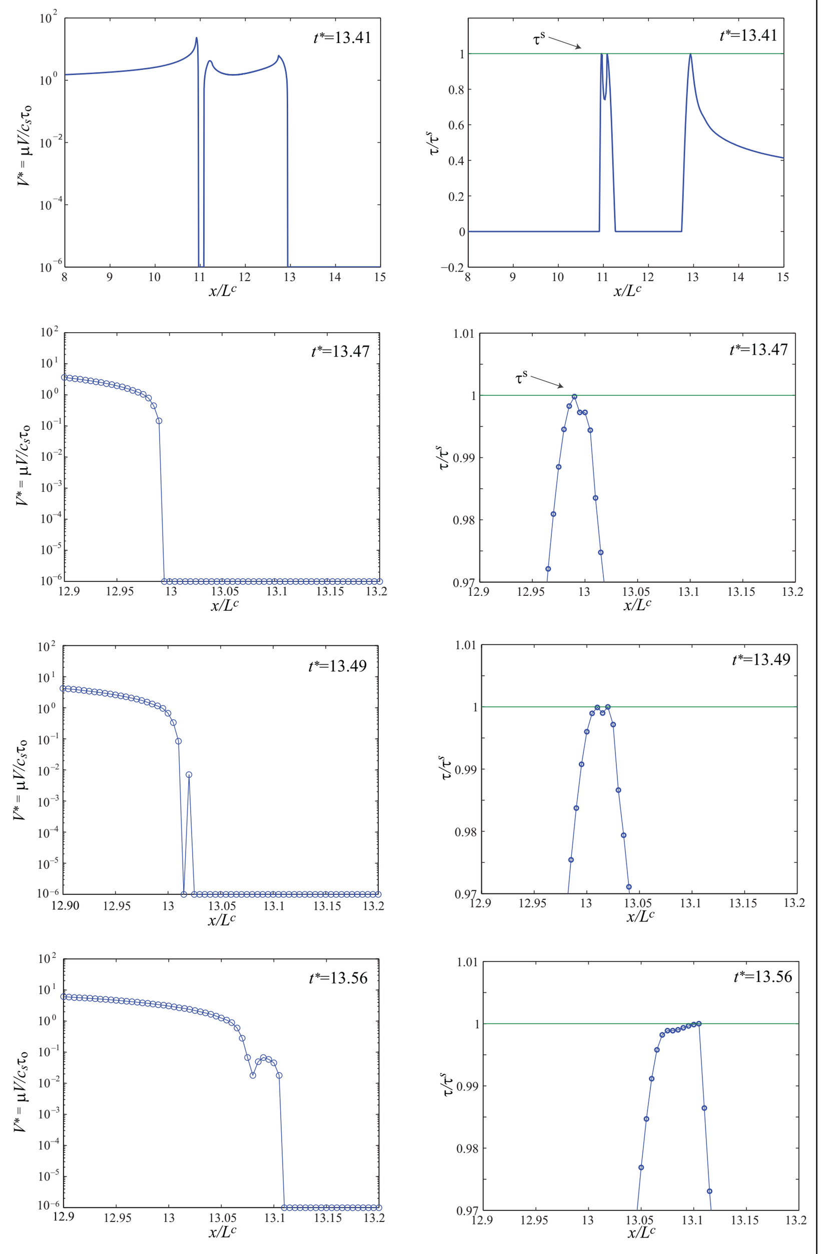
The right figure shows the rupture time on the fault for the case $N^c = L^c/\Delta x = 200, \beta = \Delta x/c_s\Delta t = 4$, where rupture time of a point is defined as the time when its sliding velocity becomes larger than 10^{-6} m/s for the first time. We observe that the secondary crack eventually accelerates to the speed larger than c_s, and the supershear is maintained despite the prestress lower than that of the Burridge-Andrews mechanism ($S = 2 > S_{crit}$).



We rerun the simulations with finer resolution (larger N^c and/or β), and observe that the supershear transition always occurs within one cell length and one time step. This implies that the rupture front abruptly jumps from Rayleigh-wave speed to a supershear speed.

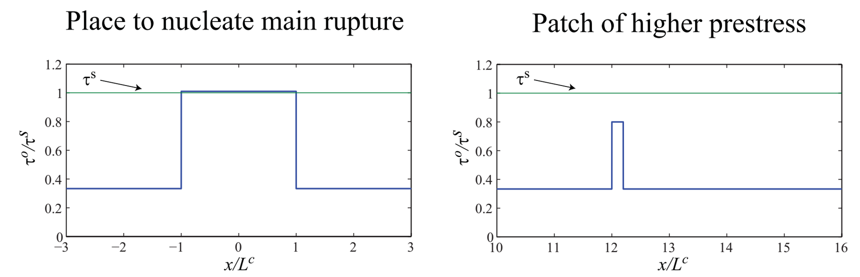


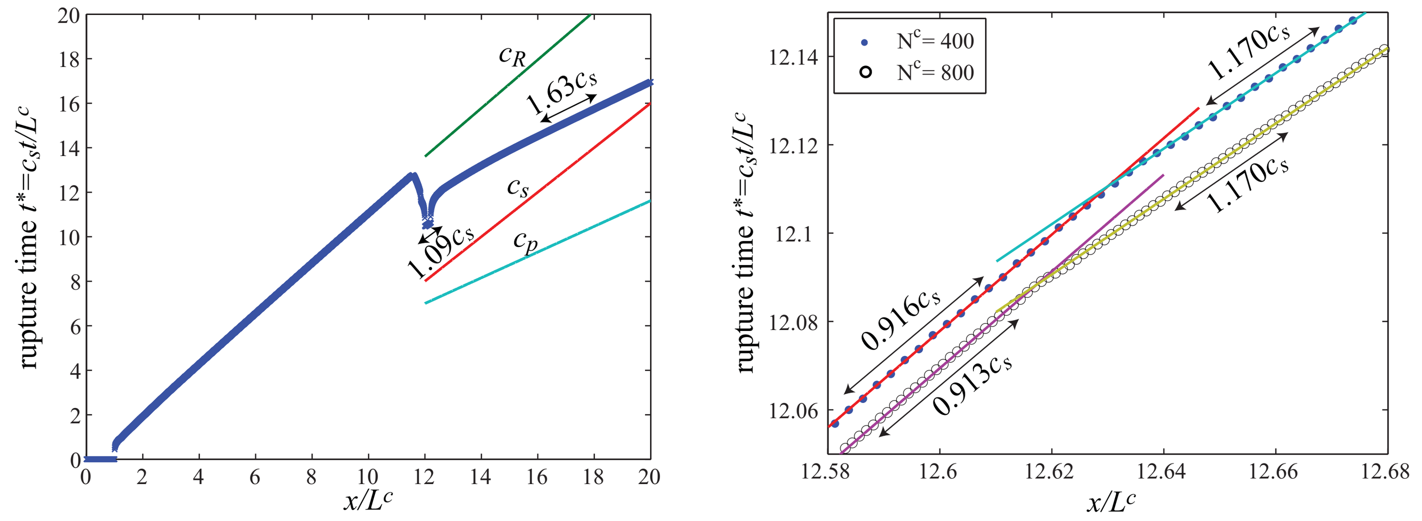
The figures on the right illustrate the transition of the secondary crack front to supershear. Snapshots of slip velocity and stress distribution on the interface are shown for $N^c = 200, \beta = 4$. (For plotting convenience, the slip velocity shown in the figures is the actual velocity plus 10^{-6}.) At time $t^* = c_st/L^c \approx 13.47$, the crack front propagates with the speed close to the Rayleigh wave speed. At $t^* = 13.49$, a daughter-like crack initiates just ONE cell ahead of the preexisting crack front, and propagates with supershear speeds immediately. This process is the same in simulations with smaller and smaller cell size Δx. Hence, in the limit of $\Delta x \to 0$, the daughter-like crack should be inseparable from the crack front, and initiate exactly at the front. This feature is fundamentally different from the Burridge-Andrews mechanism.



4.2 Advancing main rupture toward a patch of higher prestress

Instead of having a preexisting crack, we introduce a patch of higher prestress. The fault outside the patch has stress low compared to the Burridge-Andrews mechanism, with $S = 2 > S_{crit}$.

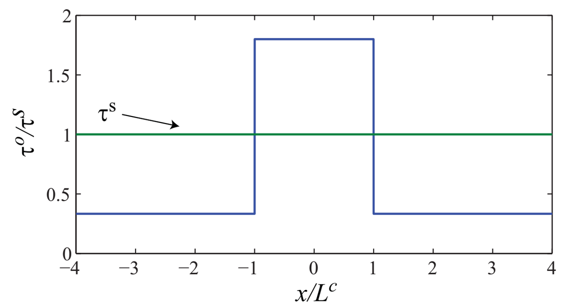




The small patch of higher prestress completely changes the rupture behavior. Without the patch, the main rupture propagates with sub-Rayleigh speeds. However, the patch (with the length $0.2L^c$ in the simulation) induces the rupture to transition to supershear, and the rupture remains supershear afterward. This shows that the level of prestress implied by the Burridge-Andrews mechanism is not needed to maintain the supershear propagation. Hence, supershear propagation on real faults can occur under prestress that are much lower than the values implied by the Burridge-Andrews mechanism.

4.3 A crack under over-stressing condition

Supershear transition can induced by overstressing a crack. Consider the case with prestress τ^o over the region $x \in [-L^c, L^c]$ set to be 0.8 times larger than the static friction strength τ^s. At the beginning of the simulation, the stress inside the patch drops from $1.8\tau^s$ to τ^s instantaneously, and spontaneous rupture propagation starts. The rupture is initially sub-Rayleigh.



An interesting phenomenon is that both our abrupt supershear transition and the Burridge-Andrews daughter crack transition appear in the simulation. First, we observe our abrupt supershear transition without initiating a daughter crack (I in the figure below). Later we observe a daughter crack that nucleates in front of the main rupture and propagates with supershear speeds (II in the figure below).

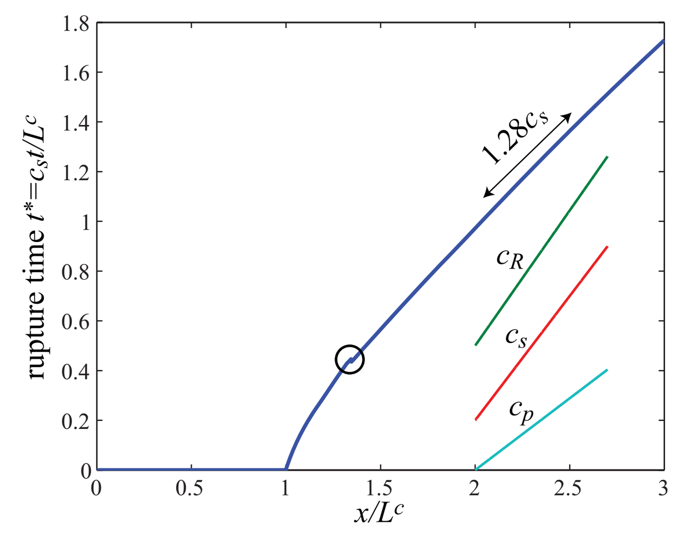


Figure: Rupture time for the case $N^c = 600, \beta = 4$.

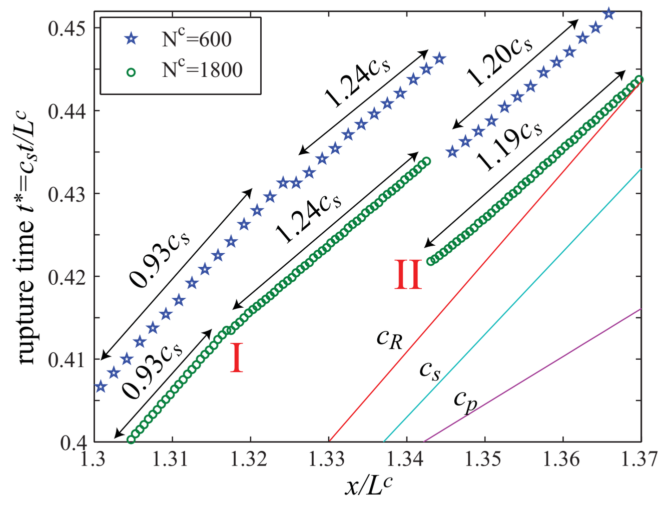


Figure: For plot convenience, we shift the rupture time downward by 0.01 for the case $N^c = 1800, \beta = 4$. Supershear transition happens twice. I denotes our abrupt transition. II denotes the Burridge-Andrews' transition.

5 Discussion

The described abrupt supershear transition mechanisms work in 3D fault models as well. We simulate the rupture propagation on a strike-slip fault interface, where a rectangular fault is surrounded by unbreakable barriers. If the model includes a patch of higher prestress or lower static friction strength, the rupture may transition to supershear speeds and the supershear can be maintained in spite of the prestress much lower than that predicted by the Burridge-Andrews mechanism. However, we notice that to trigger supershear transitions, the required patch size in the 3D strike-slip model should be much larger than that in the 2D in-plane model.

From simulations, it seems that a special loading stress τ^o is needed to cause cracks to transition to supershear. We hypothesize that the loading stress should move fast enough in the direction of the crack propagation. The most natural loading stress environment of this kind is the stress field in front of main rupture propagating with sub-Rayleigh speed advancing on a secondary crack or heterogeneity. However, there are other ways to create suitable loading stress environments. For example, we have tried to artificially impose a dynamic loading stress field $\tau^o(x,t)$ of the form $\tau^o(x,t) = f(x - c_pt)$ on a crack propagating with sub-Rayleigh speeds. We find that this also triggers supershear transition with features very similar to the preexisting crack case (4.1). Our current work is directed towards developing theoretical explanations for these phenomena.