Definitions of average stress drops for heterogeneous slip distribution: Implications to dynamic rupture process from earthquake energetics

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Abstract

An averaged stress drop is a measure of static stress redistribution and one of the fundamentally important seismological parameters. In spite of the fact that it is widely referred to in many discussions on earthquake source processes, its physically precise meaning is often obscure so that it has been difficult to develop a rigorous discussion connecting the source process and seismic observation. In the present study, we introduce several different definitions of averaged stress drops, and discuss differences between them for ruptures with heterogeneous stress drop distribution. Especially, the focus is put on the energy partitioning in earthquakes. The seismologically observed stress drop, $\overline{\Delta\sigma_M}$, is an average of stress drop distribution with the slip distribution due to uniform stress drop as a weighting function which is different from the true averaged stress drop over a ruptured area, $\Delta\sigma_A$. We show that the proper stress drop, $\overline{\Delta\sigma_E}$, to be used in the estimation of available energy is the averaged stress drop with the actual slip distribution as a weighting function. Investigation of randomized uniform stress drop models have revealed that introduction of heterogeneity and increase in the roughness of the slip distribution result in increase in $\overline{\Delta\sigma_E} / \overline{\Delta\sigma_M}$, indicating that the use of $\overline{\Delta\sigma_M}$ in the estimation of available energy causes its underestimation and thus overestimation of radiation efficiency, η_R. Note that $\overline{\Delta\sigma_M}$, as well as $\overline{\Delta\sigma_A}$, depends on the ruptured area the estimation of which has uncertainty. We propose an empirical method to approximate $\overline{\Delta\sigma_E}$ by $\overline{\Delta\sigma_M}$ by reducing the ruptured area using a threshold although it depends on the roughness of the slip distribution and overall characteristic shape of the slip distribution which probably reflects the feature of dynamic rupture propagation, such as pulse-like versus crack-like ruptures. $\overline{\Delta\sigma_E} / \overline{\Delta\sigma_M} > 1$ indicates that the previously estimated η_R (typically from 0.25 to 1) is the upper limit if there is unresolved small-scale heterogeneity. We also define the shear stress as a function of slip which represents the dynamic rupture process and comparable to $\overline{\Delta\sigma_E}$. Based on this definition, we can conclude using a physically clear logic that the scenario of the strong fault (i.e., interseismic shear stress on the fault is comparable to the static strength) with small averaged stress drop in energetic sense is not realistic.

Introduction

Seismologically estimated stress drop is a number determined for each event in a particular way [Madariaga, 1979], with the goal of capturing some average of static stress change. The estimates are used in several ways.

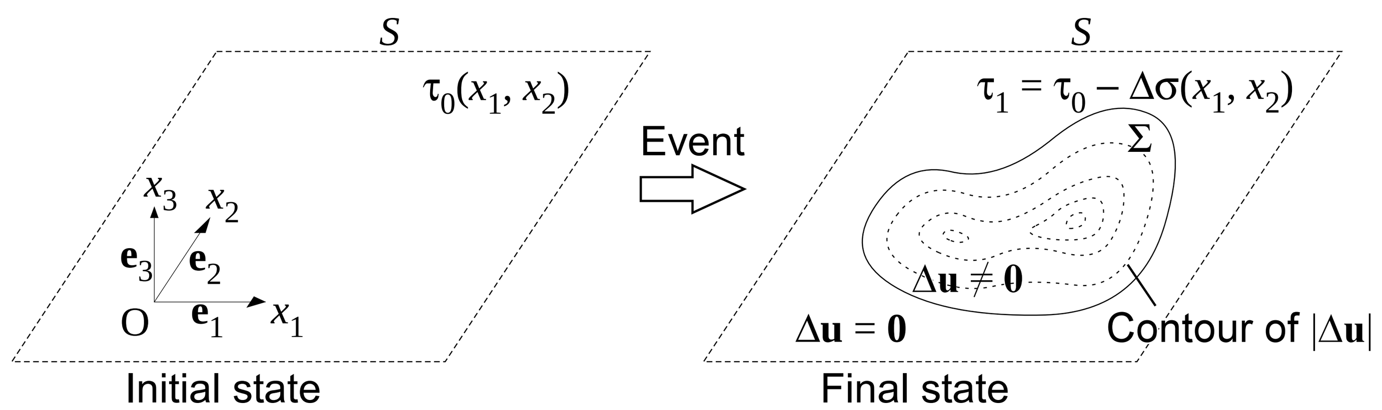
Loading rate / Stress drop = Recurrence interval ?

A typical (in an area-averaged sense) drop in the shear stress in an earthquake ?

Estimation of available energy and radiation efficiency?

Purpose of this study:
To understand the proper average for different applications, and examine the effect of heterogeneous stress drop distribution.

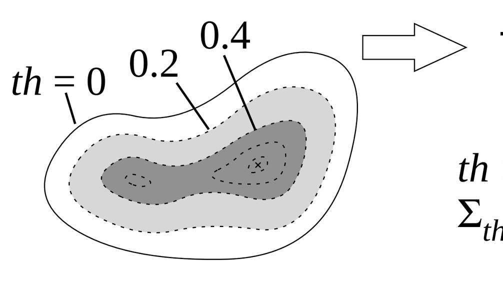
Definition of variables describing a rupture



Planar fault without opening
=> $M_{13} = M_{31}$ are only non-zero components of the seismic moment.

S : Fault plane
Σ : Ruptured area, $\Sigma = \mathrm{supp}(\mathbf{u})$
$\Delta\mathbf{u}$: Slip on the fault
τ_0 : Initial traction on a fault
τ_1 : Final traction on a fault
$\Delta\sigma$: Distribution of stress drop, $\tau_0 - \tau_1$

There is a practical difficulty in determining Σ since $\Delta\mathbf{u}$ inverted from an observation is smoothed because of band-limitedness. Also in forward simulations with a rate- and state-dependent law, for example, $\Delta\mathbf{u}$ is non-zero everywhere on the fault. This ambiguity in the definition of Σ affects averaged stress drops as shown later.



The need for definition of the ruptured area using a threshold in $|\Delta\mathbf{u}|$

th : Threshold in $|\Delta\mathbf{u}|$ as a fraction of $\max(|\Delta\mathbf{u}|)$
Σ_{th} : Area with slip higher than the threshold,
$\Sigma_{th} = \{(x_1, x_2) \mid |\Delta\mathbf{u}| > th\max(|\Delta\mathbf{u}|)\}$

Three different measures of stress drop

Seismologically estimated stress drop

$$\overline{\Delta\sigma}_M = \frac{\int_S \Delta\sigma\cdot\mathbf{E}\,dA}{\int_S \mathbf{e}_1\cdot\mathbf{E}\,dA} = c\frac{M_{13}}{|\Sigma|^{1.5}}$$

c depends on the crack model (circular, rectangular, and so on).

E: Projection of slip distribution onto $\mathbf{e}_1$ due to uniform stress drop equal to shear the modulus in the direction of $\mathbf{e}_1$. [Madariaga, 1979]

Given an uncertainty in defining Σ, $\overline{\Delta\sigma}_{Mth} = c\dfrac{M_{13}}{|\Sigma_{th}|^{1.5}}$

Average with **E** as the weight function.
Definition of "ruptured area" matters ~(Length scale)^-3

Similar averaged stress drops were used for various purposes, such as discussion on energy partitioning [Venkataraman and Kanamori, 2004].

Spatially averaged stress drop

$$\overline{\Delta\sigma}_A = \frac{\int_\Sigma \Delta\sigma\cdot\mathbf{e}_1 dA}{|\Sigma|} = \frac{\int_S \Delta\sigma\cdot\mathbf{w}\,dA}{\int_S \mathbf{e}_1\cdot\mathbf{w}\,dA} \qquad \mathbf{w} = \begin{cases}\mathbf{0} & \text{Outside } \Sigma\\ \mathbf{e}_1 & \text{Inside } \Sigma\end{cases}$$

Given an uncertainty in defining Σ,

$$\overline{\Delta\sigma}_{Ath} = \frac{\int_{\Sigma_{th}} \Delta\sigma\cdot\mathbf{e}_1 dA}{|\Sigma_{th}|} = \frac{\int_S \Delta\sigma\cdot\mathbf{w}_{th}dA}{\int_S \mathbf{e}_1\cdot\mathbf{w}_{th}dA}$$

$|\Sigma_{th}|$: Area of Σ_{th}

$$\mathbf{w}_{th} = \begin{cases}\mathbf{0} & |\mathbf{u}(x_1,x_2)| \le th\max(|\mathbf{u}|)\\ \mathbf{e}_1 & \text{otherwise}\end{cases}$$

Average with a boxcar function as the weight function.
Definition of "ruptured area" matters (as $\Sigma \to S$, $\Delta\sigma_A \to 0$)
$\overline{\Delta\sigma_A}$ is different from $\overline{\Delta\sigma_M}$ in general [Madariaga,1979].

Energy-based averaged stress drop

Continuous series of static solutions connecting the initial and the final states:

$$\Delta\mathbf{u}_\nu = \lambda\Delta\mathbf{u} \quad \text{and} \quad \tau_\nu = \tau_1 + \lambda\Delta\sigma$$

The change in the strain energy (potential) is given by

$$\Delta W = \int_S\int_{\text{initial}}^{\text{final}} \tau_\nu\cdot d\Delta\mathbf{u}_\nu dA = \int_S\int_0^1 \tau_\nu\cdot\Delta\mathbf{u}\,d\lambda dA = \int_S \tau_1\cdot\Delta\mathbf{u}dA + \frac{1}{2}\int_S \Delta\sigma\cdot\Delta\mathbf{u}dA$$

Dissipation if the fault slips at the final traction; ΔW_0, "available energy"

In application,

$$\Delta W_0 = \frac{1}{2}\overline{\Delta\sigma}_E\int_S \mathbf{e}_1\cdot\mathbf{u}dA \qquad \text{Therefore,} \qquad \overline{\Delta\sigma}_E = \frac{\int_S \Delta\sigma\cdot\mathbf{u}dA}{\int_S \mathbf{e}_1\cdot\mathbf{u}dA}$$

Average with $\Delta\mathbf{u}$ as a weight function.
Definition of "ruptured area" does not matter.
$\overline{\Delta\sigma_E}$ is the proper value to be used in estimating ΔW_0.

> Those 3 different measures match if $\Delta\sigma$ is uniform.
> ***How about in heterogeneous cases?***

Random 2D slip distribution

$$\Delta u_1 = C_n\,\phi\,f_{dev}\left(F^{-1}\left[H(128\pi - k)k^{-\frac{\nu+1}{2}}F\left[R(x_1,x_2)\right]\right]\right);\; u_2 = 0$$

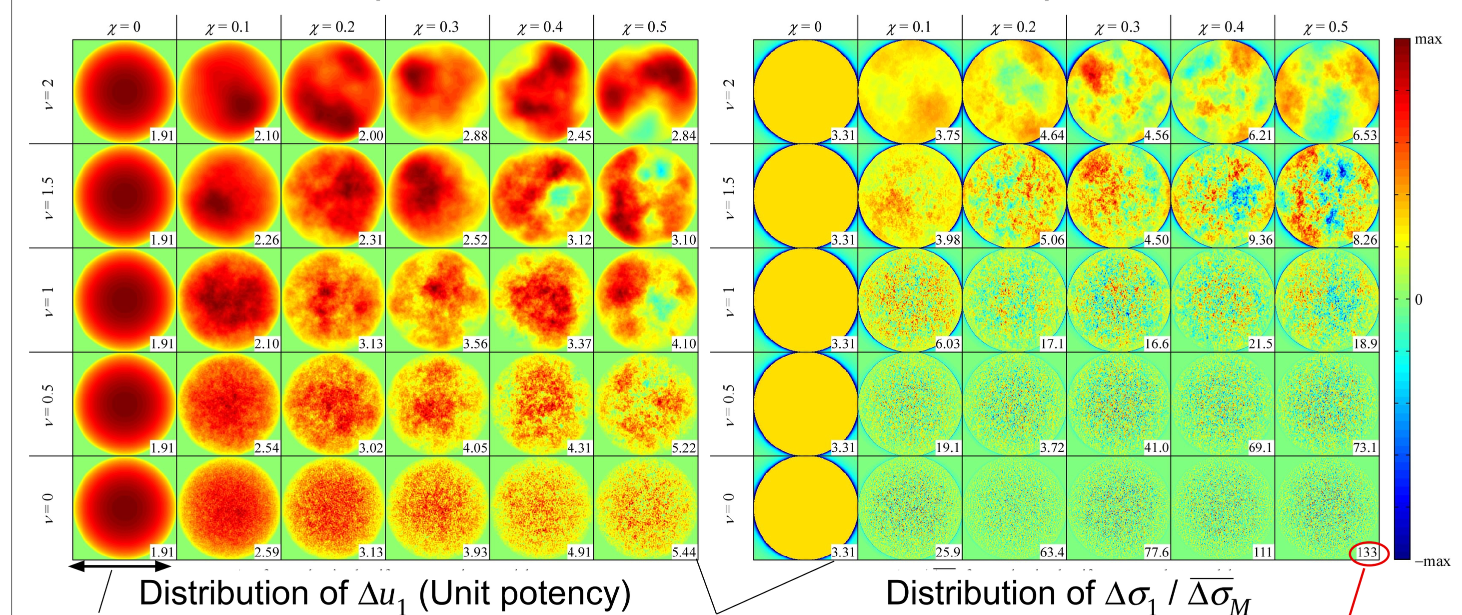
Truncation in the wavenumber. Filtering, ν: scaling exponent ~ 1. Gaussian random field. Normalization. Unperturbed slip distribution. Adjustment of χ = (standard deviation) / (mean). 2D Fourier transformation. 2D inverse Fourier transformation.

[Motivated by Lavallée et al., 2006]

Δu_1 is generated by randomizing several different slip distributions, ϕ.
Σ is a circle of unit diameter, and the total potency is unity.

Effect of roughness of the slip distribution

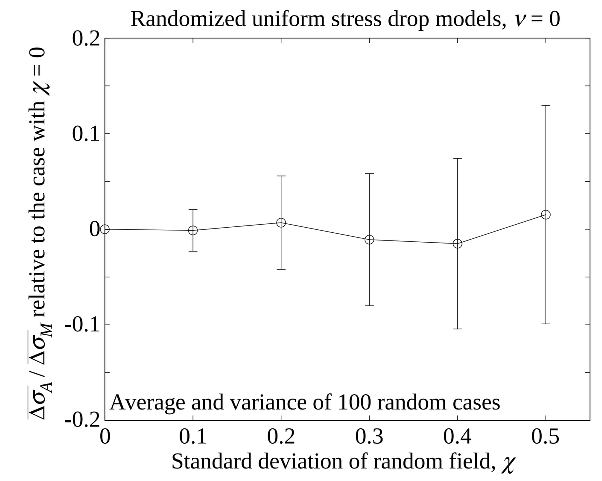
Example of randomized uniform stress drop models



$\overline{\Delta\sigma_A}$ vs. $\overline{\Delta\sigma_M}$

The randomization taken here does not affect $\overline{\Delta\sigma_A}$ systematically.

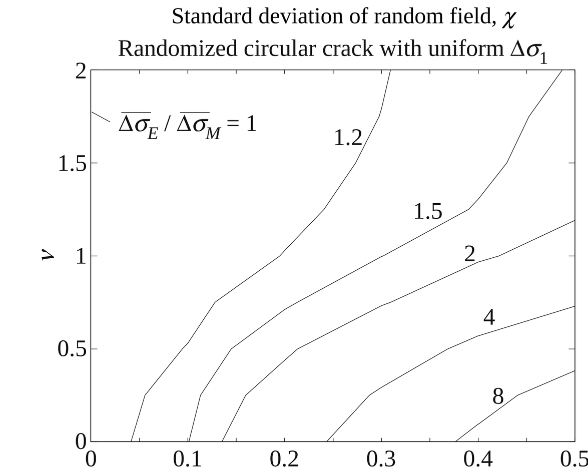
Although the local perturbation in $\Delta\sigma_1$ is larger than $\overline{\Delta\sigma_M}$ by more than 2 orders of magnitude in the most rough case ($\nu = 0$, $\chi = 0.5$) the variance in $\overline{\Delta\sigma_A}$ is only around 0.1.



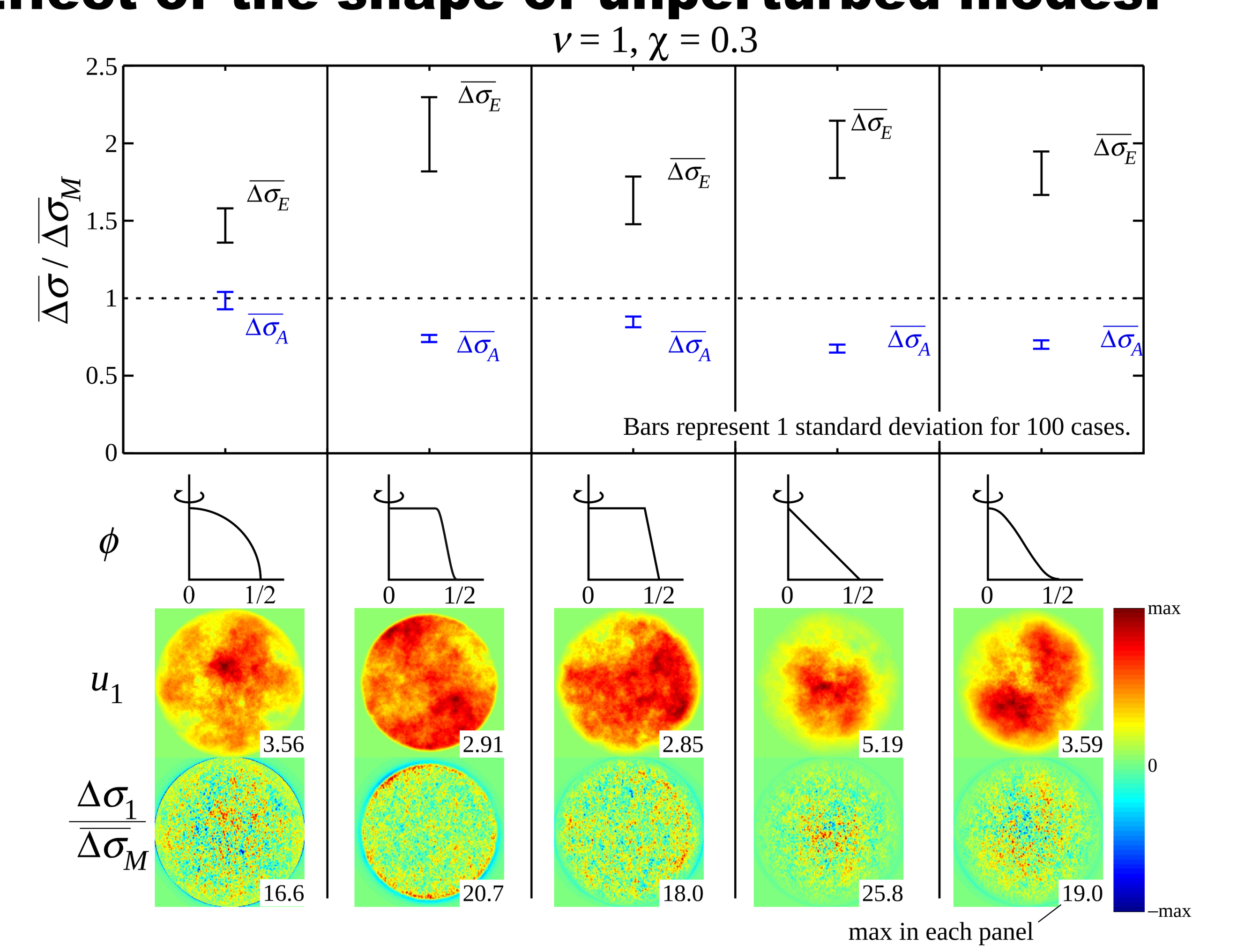
$\overline{\Delta\sigma_E}$ vs. $\overline{\Delta\sigma_M}$

Introduction of heterogeneity and an increase in the roughness (a decrease in ν and an increase in χ) always causes an increase in $\overline{\Delta\sigma_E} / \overline{\Delta\sigma_M}$.

The use of $\overline{\Delta\sigma_M}$ in an estimation of ΔW_0 causes its underestimation.



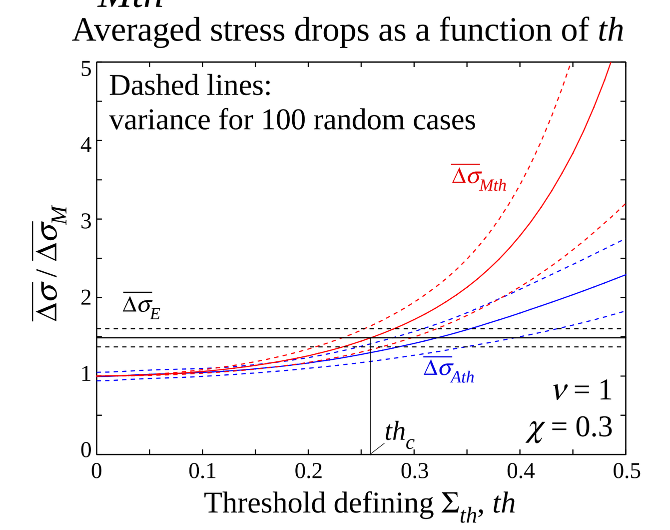
Effect of the shape of unperturbed modesl

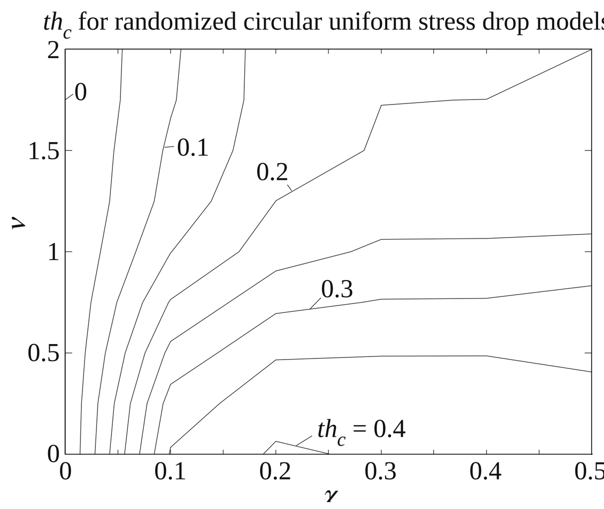


The model before the randomization causes systematic difference between $\overline{\Delta\sigma_M}$ and $\overline{\Delta\sigma_A}$. Typically, $\overline{\Delta\sigma_E} > \overline{\Delta\sigma_M} > \overline{\Delta\sigma_A}$ in the cases studied.

Approximation of $\overline{\Delta\sigma_E}$ by $\overline{\Delta\sigma_{Mth}}$

Trimming of Σ by a proper threshold [e.g., Somerville et al., 1999, Venkataraman and Kanamori, 2004] increases $\overline{\Delta\sigma_{Mth}}$ and improves estimation of $\overline{\Delta\sigma_E}$ and thus ΔW_0.





The critical value of th, th_c, depends on the roughness of the random field, as well as the shape of the unperturbed model. For the randomized circular uniform stress drop models, $th \sim 0.2$ may be a reasonable selection.

Implications for dynamic rupture process

Venkataraman and Kanamori [2004] estimated the radiation efficiency, η_R, using seismologically estimated stress drops, and concluded that η_R is not much larger than unity. Our present study suggests that unresolved small-scale heterogeneity leads to overestimation of η_R, and thus the about η_R conclusion is reconfirmed even considering observational limitation.

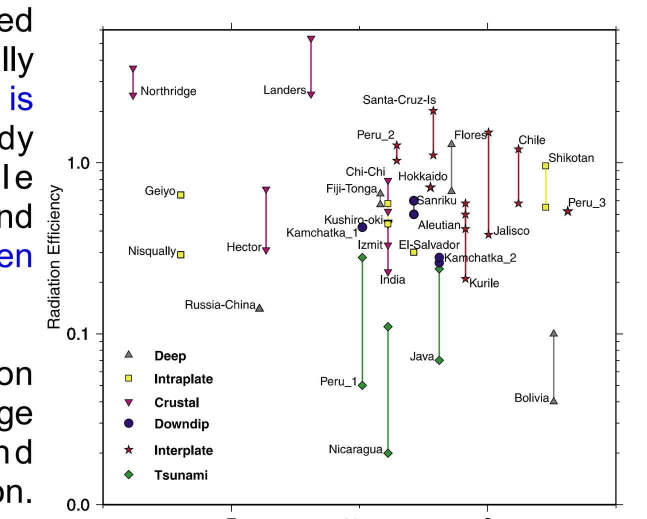
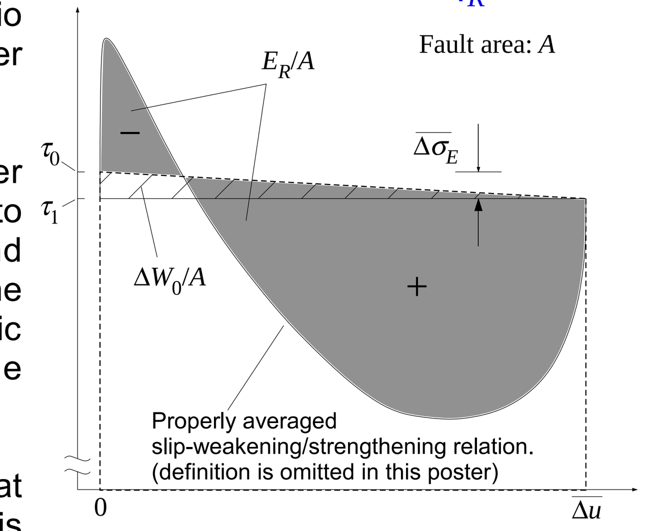


Figure from Venkataraman and Kanamori, 2004

Recent experimental studies on rock friction [e.g., Sone and Shimamoto, 2009] reported large strength drop at coseismic slip rate and comparable restrengthening during deceleration. They tried to argue rather small stress drop in an event assuming the initial and final stress is given by the shear stresses at the initiation and termination of slip, respectively. Such a scenario is unrealistic because it implies η_R much larger than unity.

Unrealistic scenario: $\eta_R >> 1$



There are potentially plausible end member scenarios: [1] The initial shear stress is close to the static friction everywhere on the fault, and $\overline{\Delta\sigma_E}$ is comparable to the strength drop; [2] The initial shear stress is much smaller than the static friction, and $\overline{\Delta\sigma_E}$ is much smaller than the strength drop.

If [1] is the case, the well-established fact that the seismologically obtained stress drop, $\overline{\Delta\sigma_M}$, is much smaller than the normal stress at depth requires either much smaller strength drop than the normal stress (e.g. due to high ambient pore pressure) or significantly larger $\overline{\Delta\sigma_E}$ than $\overline{\Delta\sigma_M}$ which indicates fairly rough slip distribution. Also, the ruptures would propagate in a crack-like manner because of high initial shear stress.

[2] is the "statically strong, but dynamically weak fault" model [e.g., Lapusta and Rice, 2003]. Pulse-like rupture propagation is more likely in [2] than [1].

Plausible end member scenarios