Bayesian modeling of the under constrained 7.8 Tohoku-Oki aftershock

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Abstract

We used a fully Bayesian probabilistic formalism requiring no a priori spatial regularization to invert the slip distribution of the 7.8 Tohoku-Oki aftershock from the very dense Japan GPS network. We extracted the static surface displacement from 5 min high rate GPS times series and we considered a one dimension elastic structure. This inverse problem is highly under constrained. We show here the slip distribution obtained and different diagnostic parameters such as the probability density function (pdf) of each parameter or a posterior covariance.

Introduction

The 9.0 Tohoku-Oki earthquake, occurred on March 11 2011, was the best ever recorded event of this magnitude. Here we focus on the magnitude 7.8 aftershock happened 28 minutes after the main event. This event is particularly interesting because of its location; historically many quakes happened near the coast of Japan at the boundary between the Pacific and the Eurasian plates but none of them near the latitude of 36 degrees (see figure 1). This aftershock allows us to investigate a poorly known area where the deficit of seismic and inter seismic deformation could lead to a possible future large event. We used high rate GPS data from the Geodetic Survey of Japan (GSI) and a fully Bayesian probabilistic formalism requiring no a priori spatial regularization to model the slip distribution along the fault responsible for this aftershock. Even though the data network is particularly dense, its spatial configuration makes the inverse problem highly under constrained. This aftershock modeling was an opportunity to test how much information we can obtain from such a problem with a Bayesian approach.

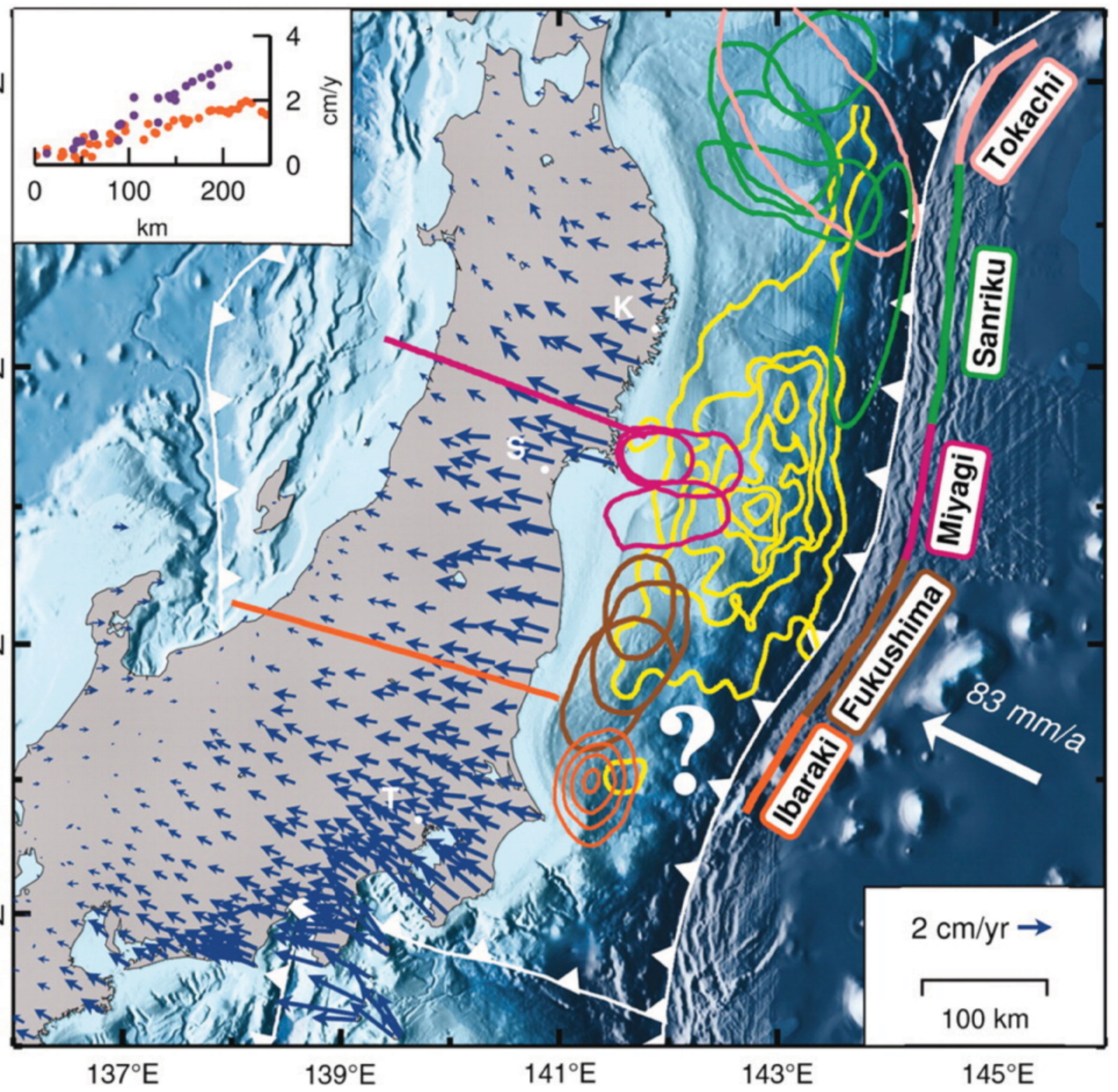

Figure 1: Hisorical Mw>7.0 earthquakes offshore Northern and Central Japan and secular interseismic surface deformation (blue arrays, and cross sections in the top left box). Figure form (1)

A Bayesian probabilistic formalism

We used a code named CATMIP, for Cascading Adaptive Temporal Metropolis In Parallel (from (2)), to explore the range of likely fault slip models. Instead of inverting the matrix of the Green's function, the code test a high number of possible slip model that fit the data the best taking into account the error on the data (well known) and the error on the Green' functions (unknown, estimated by the code). At the end, we obtain a probability density function (pdf) of each fault patch parameters, for the strike slip and the dip slip component.

$$P(m|d) = P(m)P(d|m) \qquad (1)$$

$$P(m|d) = Uexp(-\frac{1}{2}(d-Gm)^{T}Cd^{-1}(d-Gm)) \qquad (2)$$

The equation governing the algorithm is the Bayes theorem (1). Then, d being the data, m the slip model, G the Green's functions and Cd the covariance on the data, we derive (2) from (1) considering a Gaussian distribution of the likelihood of d and a uniform prior model U=P(m).

Data

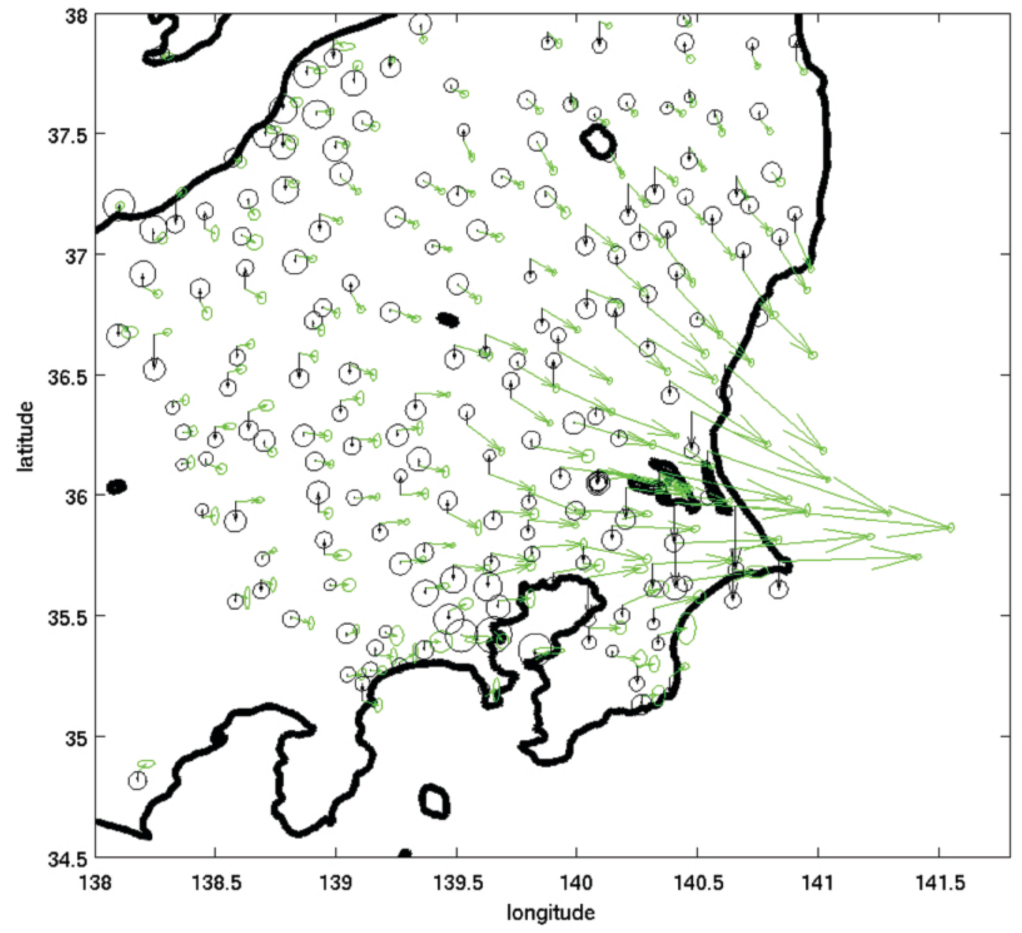

Figure 2: horizontal (green) and vertical (blue) data and their error ellipses derived form 5min high rate GPS time series.

We derived the static displacement (shown in figure 2) at the surface from 5 min high rate time series. Since the aftershock occurred only 28 min after the main event, there is possibly some long period signal remained. Even if this network is very dense, its configuration is far from optimum. The entire network is west of the aftershock. So, the inversion is "east blind": the resolution decreases with the longitude. This configuration makes the problem highly under constrained.

Slip model

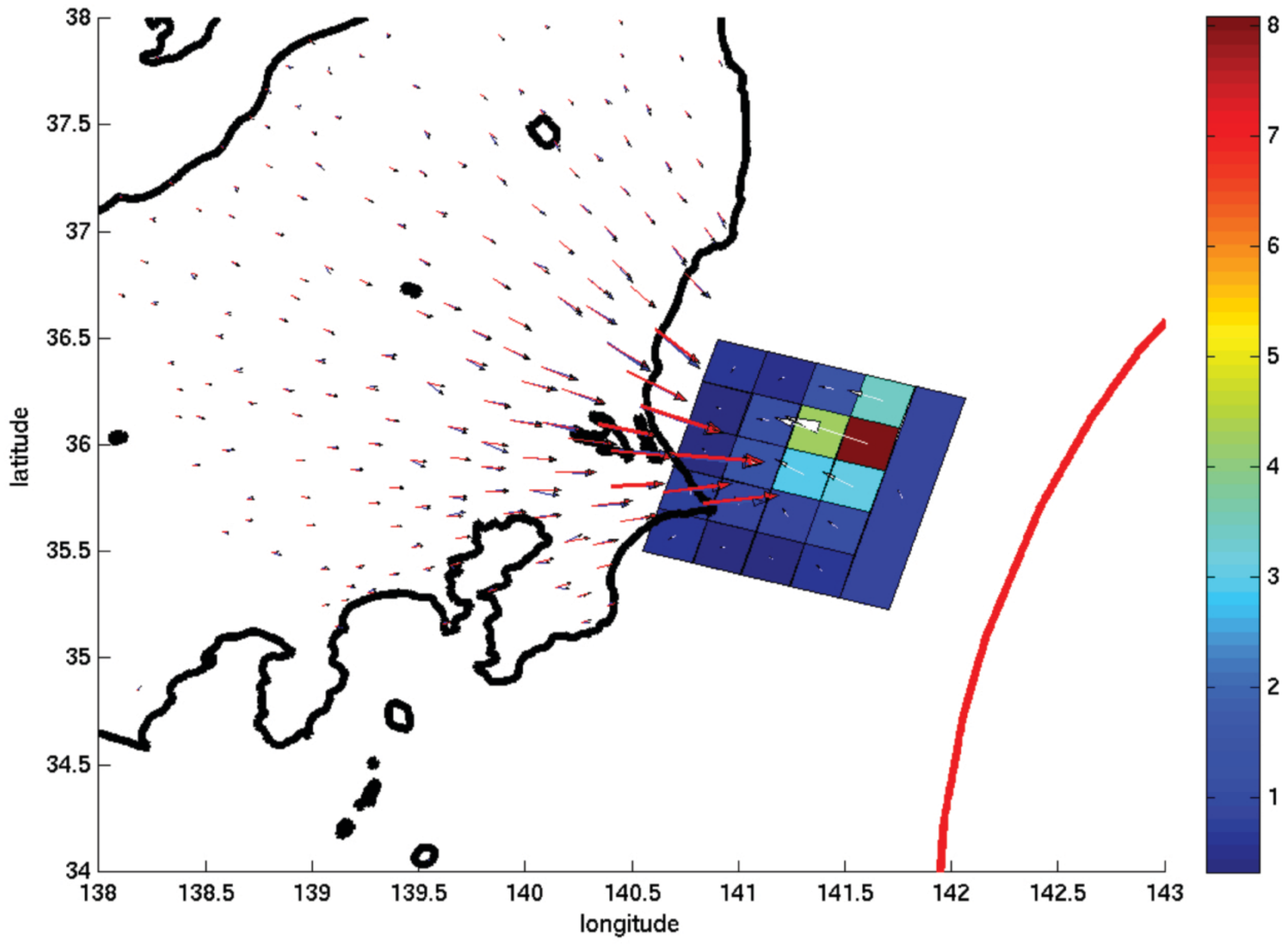

Figure 4: slip model associated with the 7.8 2011/03/11 Tohoku-Oki aftershock and data fit. The blue arrows are the data, the red are the predicted data, the red line is the trench. The model assumes the layered media shown in figure 3.

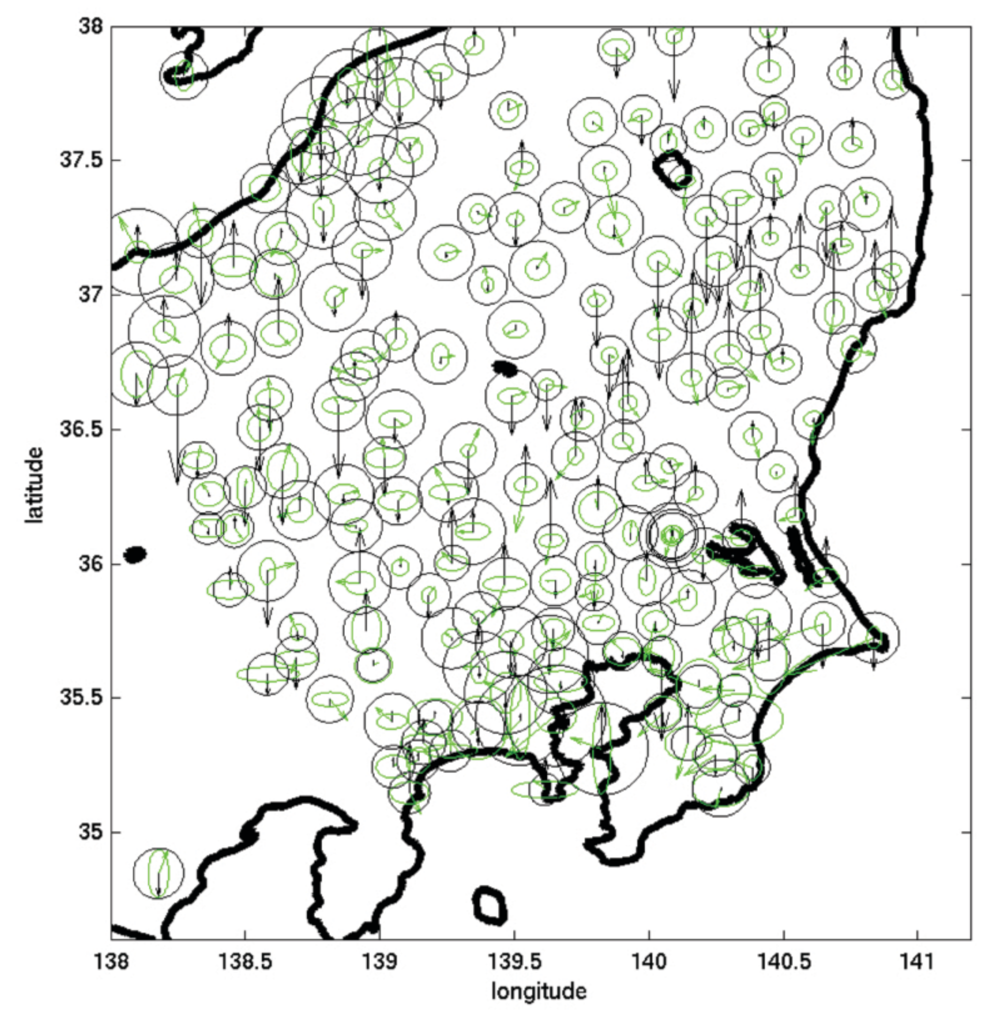

Figure 5: horizontal (green) and vertical (black) residual of the slip model shown in figure 4 with ellipses of data uncertainties. We see no coherent signal on the horizontal and an amplitude below the level of noise. The vertical shows a higher misfit.

We considered a curved fault geometry interpolating the regional seismic activity over a surface. The strike slip and dip slip components of 21 patches were inverted from the static data, 4 lines of 5 patches (23X23 km) and one big patch (23X115 km) near the trench for covariance issues. The amplitude and the direction of the slip on each patch are plotted in figure 4. The slip amplitude values are the means of the whole set of model tested by the algorithm.

The epicenter is found at longitude 141.6 and latitude 36.0, the depth at 22 km, the magnitude found is 7.8. Since the vertical do not play an important rule in the inversion due to their uncertainties and inaccuracies (see figure 6), the depth is mostly controlled by the fault geometry.

The residual show a low amplitude incoherent signal (see figure 5). However, the maximum slip is found in an area where the resolution is presumably low, so the uncertainties are important and hard to estimate.

Elastic structure

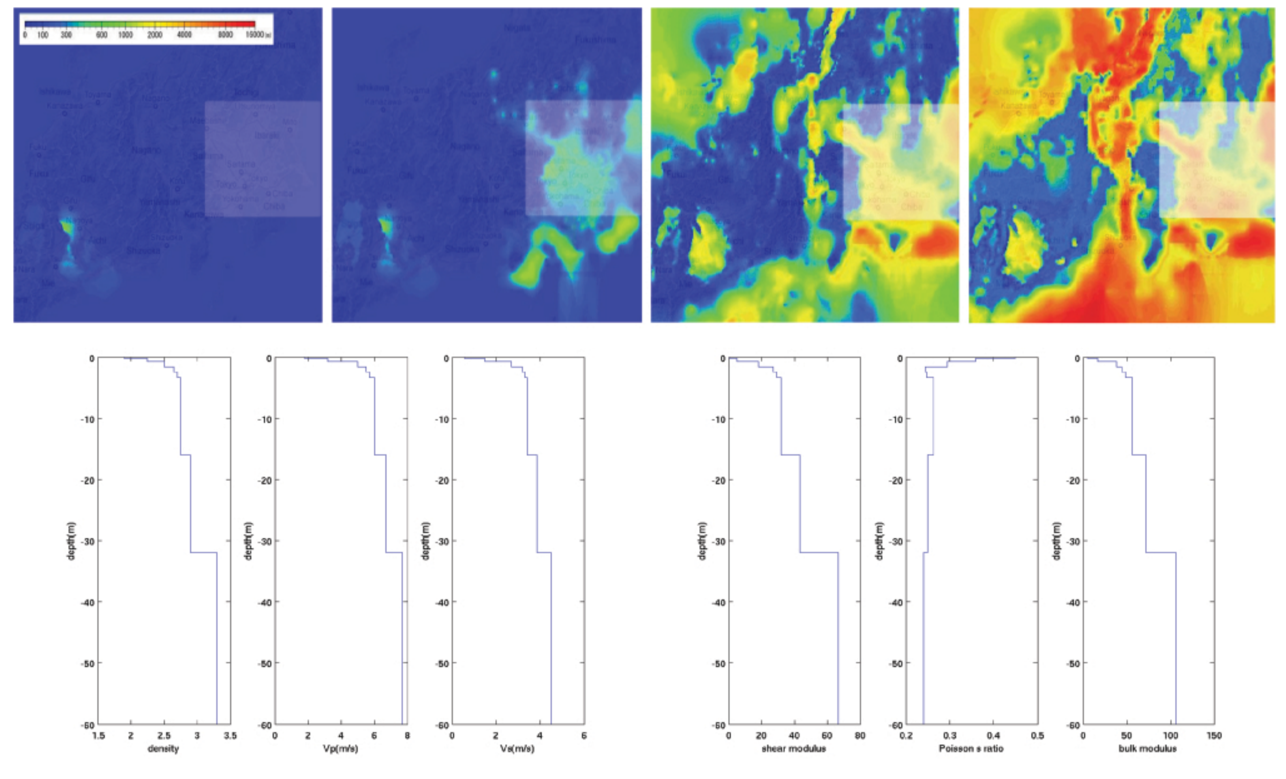

Figure 3: up: depth of the Vs = 550, 950, 1600 and 2900 m/s iso-surfaces (from (3)) integrated over the strongest data location area (transparent squares) to characterize the elastic structure of the shallowest part. The deepest part was made by the same process from the 3D structure in (4). Bottom: resulting density, Vp, Vs, shear modulus, Poisson's ratio and bulk modulus profiles.

We considered a one dimensional layered space in our model. Since the waves we record comes from the fault at a certain depth to the GPS stations at the surface; we built our one 1D model form an offshore tomography for the deep part and an onshore tomography for the shallow part. The deep part of our profile is from (4). We built the shallow part by integrating the 3D tomography from (3) over the two horizontal dimensions of the strongest data area (see figure 3). Since the resolution of a tomography decreases with depth, the precision of our profile is much better in the first kilometers of depth.

Model error estimation

At some point, the change on the amplitude of the slip model needed to better fit the data is too important compare to the gain on the fit itself. We can estimate a critical value for each model and get a pdf of the estimated model error (see Figure 6).

We see that the model error on the vertical data is huge compare to the horizontal. In response to this estimation, the algorithm doesn't try to fit the vertical data as well as the horizontal and so the vertical resolution is very coarse.

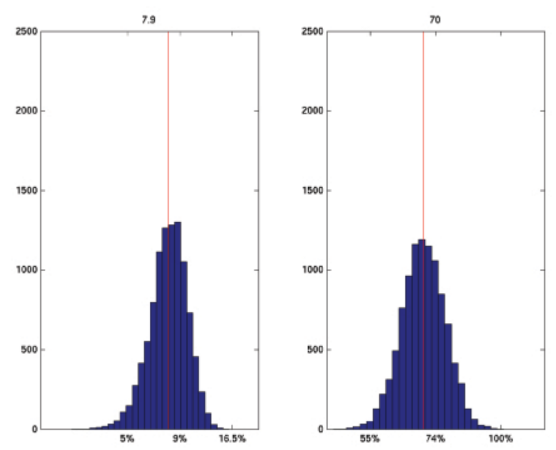

Figure 6: pdfs of the horizontal (left) and vertical (right) estimated error in percentage of the modeled slip. The red line are the means. The values at the top are the means of the distributions.

Likelihoods and Covariances of the model parameters

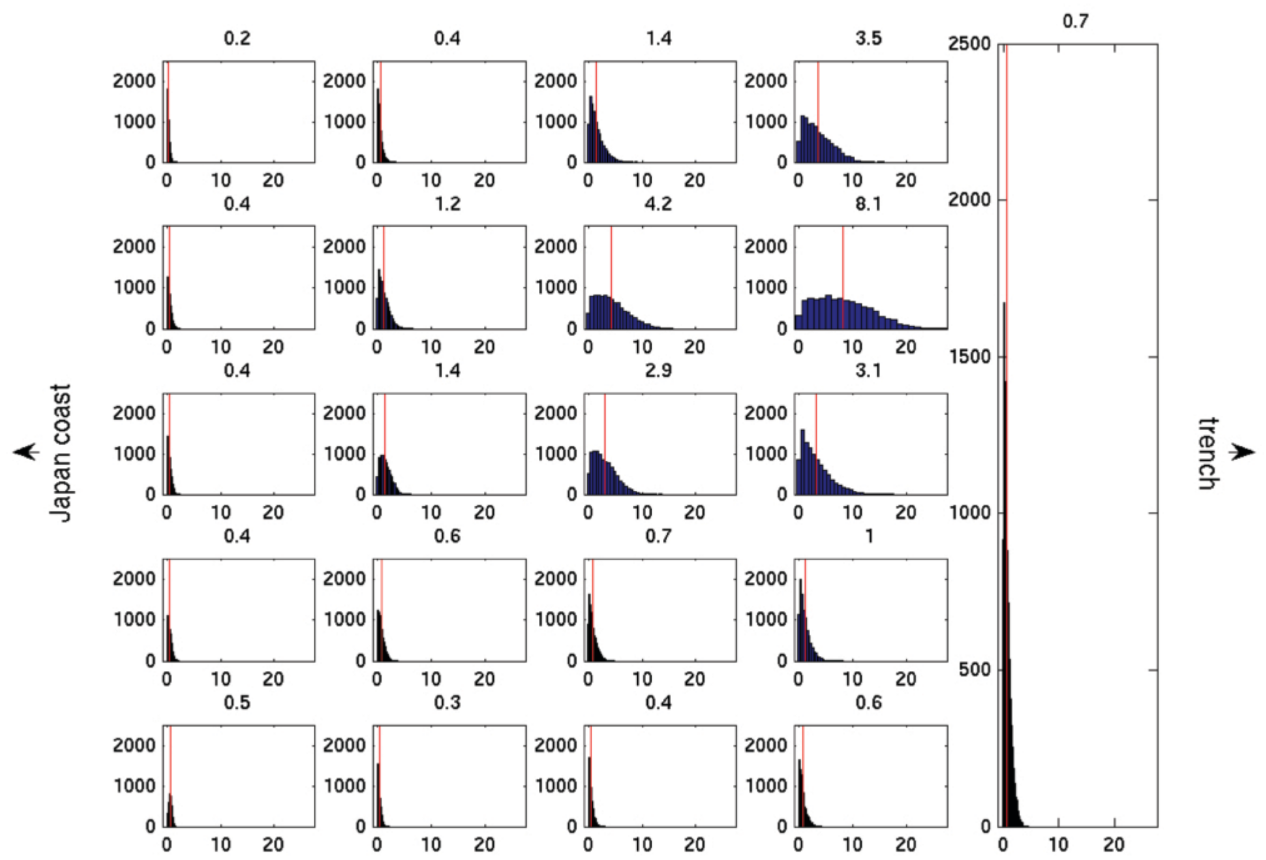

Figure 7: posterior pdfs of the dip slip component of the slip on each patch. The abscissa is the amplitude of the slip in the dip slip direction, the ordinate is the corresponding number of model samples. The red lines show the mean values, and these mean values are written at the top of each histogram. These pdfs can be assimilated to a likelihood.

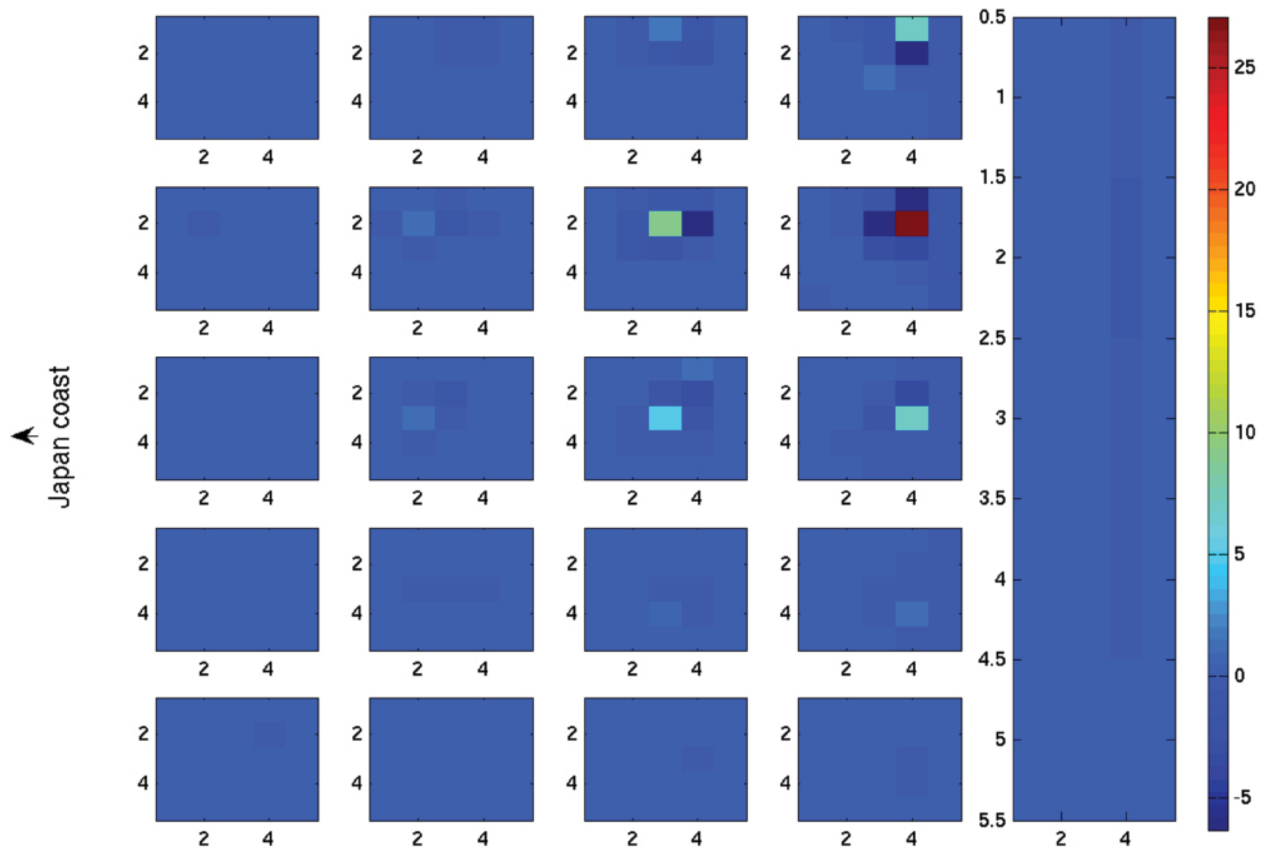

Figure 8: posterior covariance of each patch with the others for the dip slip component. This covariance is calculated a posteriori from the pdfs shown in figure 7.

Since we do not use any a priori spatial regularization, we cannot calculate a resolution from usual methods. However, the information contained in the a posteriori pdfs of each parameter (shown in figure 7) gives us a lot of information. The fully resolved case would be a Dirac function. As we starts form a prior uniform model with a Gaussian magnitude, we expect Gaussian pdfs with a mean and a standard deviation. The mean is the best fitting value over the explored model space and the standard deviation tells us about the resolution and the covariances. We observed Dirichlet shapes (functions starting by a pic at 0 and then exponentially decreasing) on the poorly resolved patches. They are the consequences of a high covariance.

We can calculate an a posteriori covariance Cm from these pdfs (see figure 8) from Cm (X,Y) = E((X-E(X)) (Y-E(Y))), E being the expectation. It shows us that the patches anti covary with their neighbors. As expected, the covariance increases in the east direction, and increasing the size of a patch reduces its covariance.

Conclusion

We found a static slip distribution of the 7.8 Tohoku aftershock using GPS data and a Bayesian probabilistic formalism with no a priori spatial regularization in a layered space. We tried to find a metric to discriminate what we can resolve and what we cannot using the information contained in the pdfs. Their shapes, their standard deviations and their covariances seem to give coherent information. Further work will be necessary in order to define an absolute metric of the true information content on a Bayesian frame.

References

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