



# 3D Finite Element modeling of Precariously Balanced Rocks

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## Introduction

Many bands of precariously balanced rocks have been discovered in Southern California by Brune et al. Since these rocks have been precariously placed for thousands of years, they can help in providing a constraint on the maximum ground shaking experienced by that region during the age of the PBR.

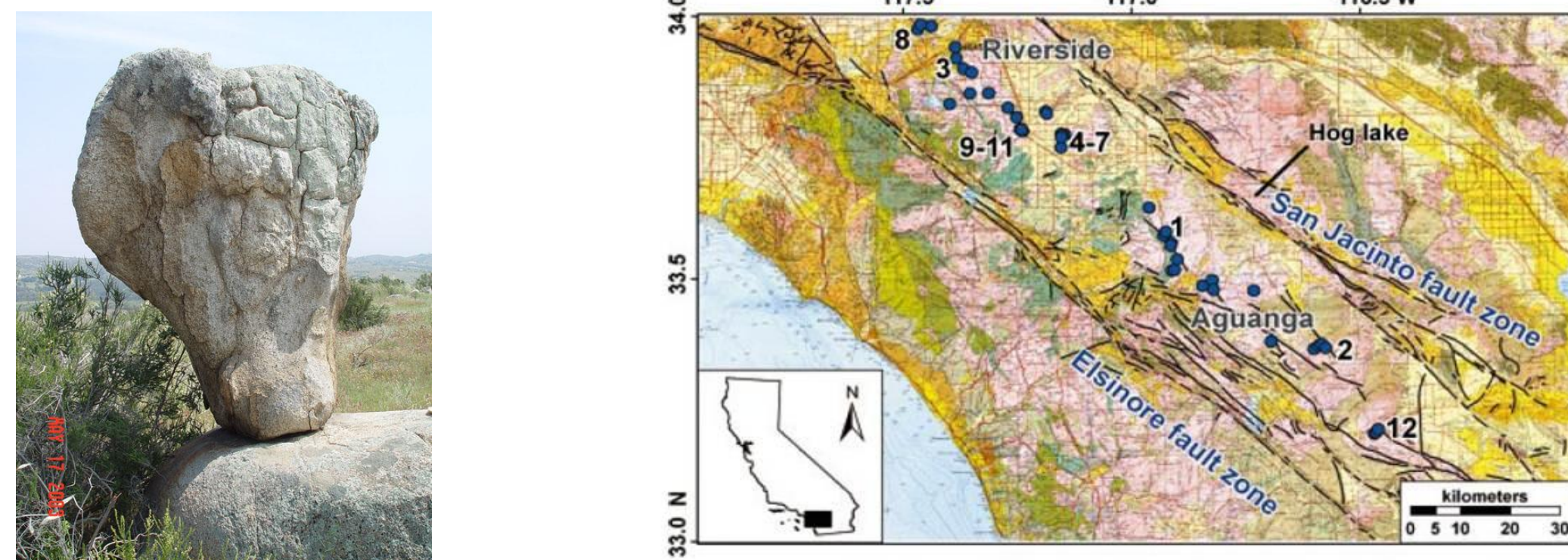


Figure1 (a) Benton rock and (b)band of precariously balanced rocks between Elsinore and San Jacinto fault zone (Brune et al)

We are trying to build a 3D finite element model of the rock in order to understand the complicated 3D response of the rock. To establish the proof of concept, we are modeling the Echo Cliff PBR which is located in the Western Santa Monica mountains.

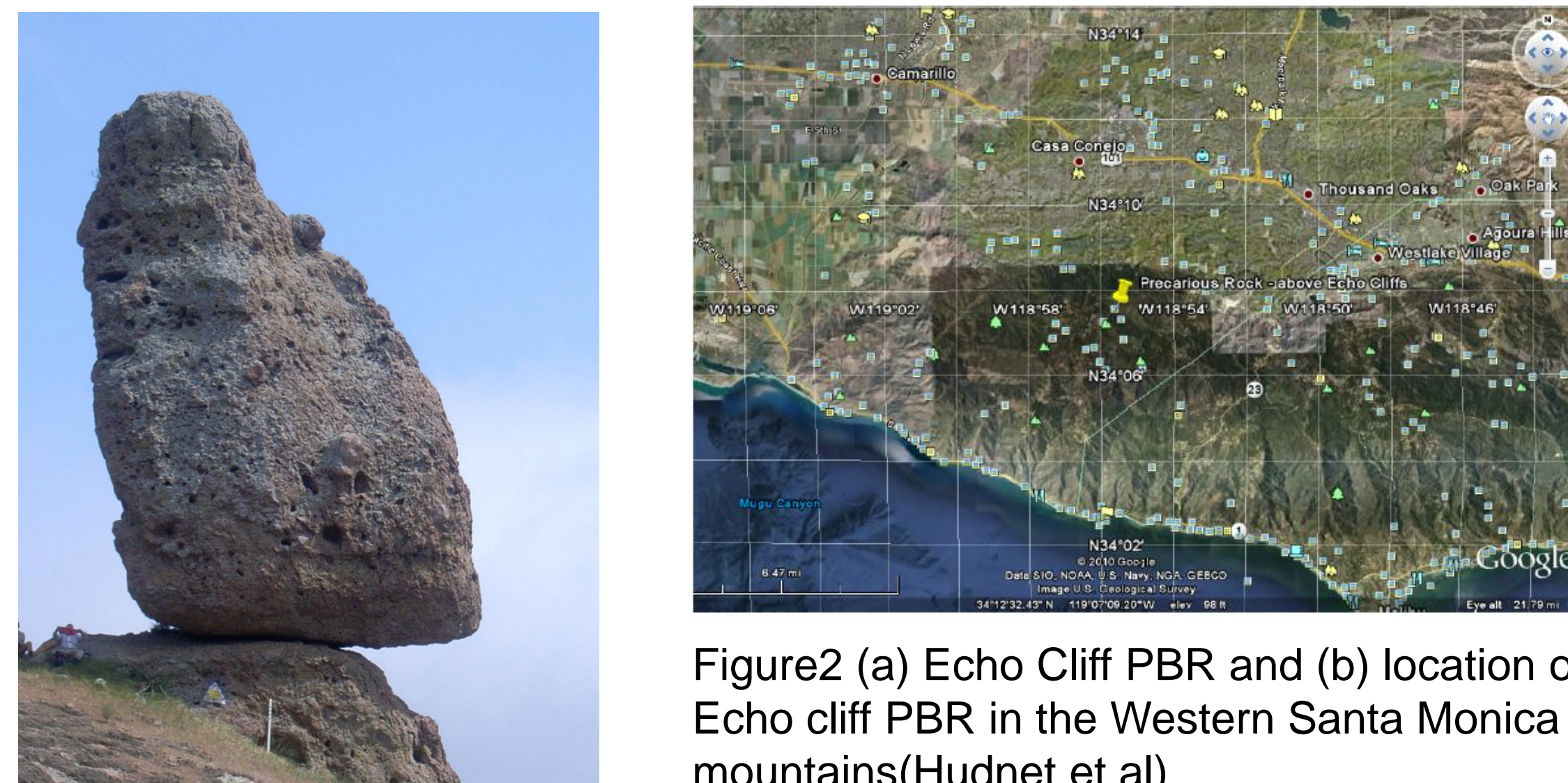


Figure2 (a) Echo Cliff PBR and (b) location of Echo cliff PBR in the Western Santa Monica mountains(Hudnet et al)

The ultimate goal is to arrive at probabilistic constraints on region-wide ground shaking intensity by combining the results of this study with cosmogenic dating of these rocks.

## Past Work

Housner was amongst the first to analyze the rocking response of structures. He derived a theoretical model for the dynamic response of a rigid rectangular block on a rigid ground. This has been the basis for many PBR related studies. Some analysis have been conducted on PBRs by assuming the 3D rock with complex basal geometries can be reduced to a simple 2D model with 2 point contact system, similar to Housner's block.

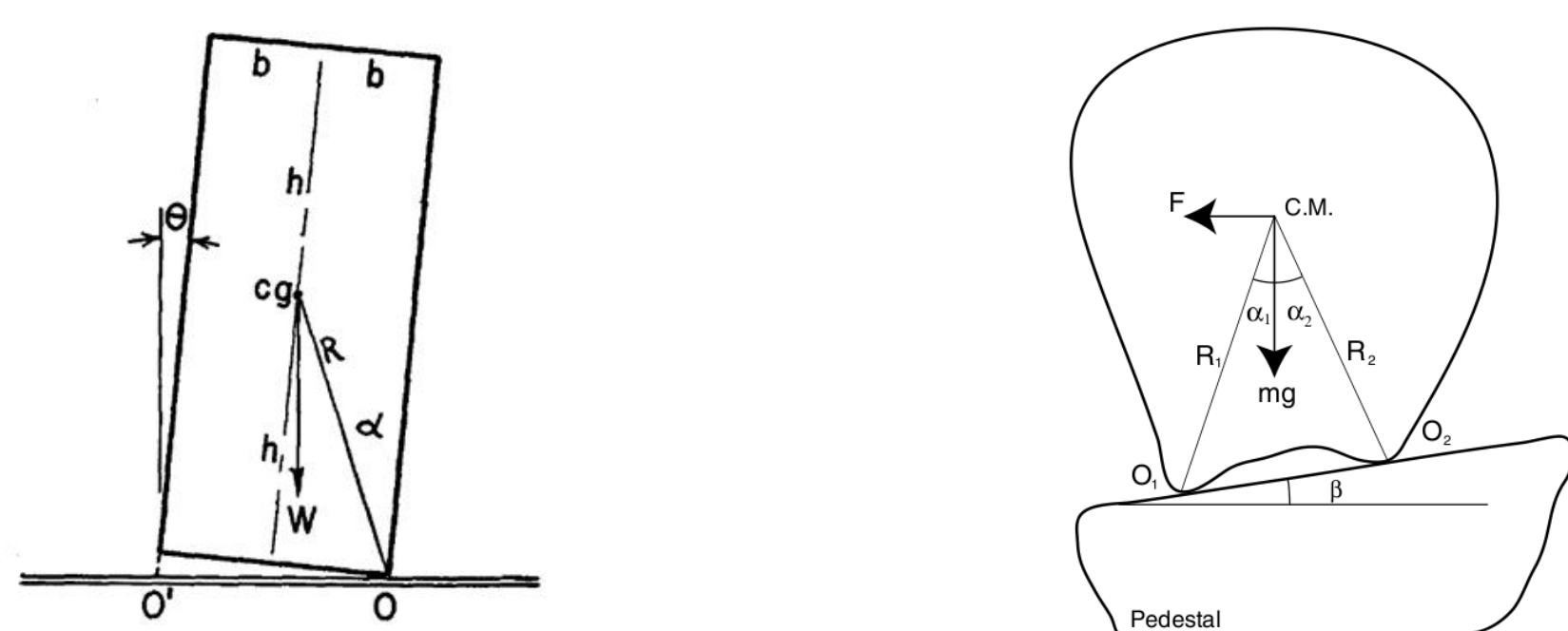


Figure3 (a) Housner's block and (b) Simple 2D approximation of a rock by a 2 contact point system

Shake table experiments have been conducted on some rocks subjected to real ground motion records in order to obtain the overturning probability of the rock. Field tests have also been conducted on some rocks to obtain the quasi static toppling acceleration of the rock.

### Need for 3D FEM model:

Most of the rocks have a very complex and asymmetric shape and hence the motion of the rock will not be restricted to a plane. A 3D Finite element model with very accurate basal configuration will help in capturing the 3D response of the rock.

## Effect of Geometry on the dynamics of rocking bodies

Before moving on to the actual rock, we would like to test the effects of geometric parameters, mainly the slenderness and the basal shape factor, on the response. So we are using a 2D idealized model with a curved base.

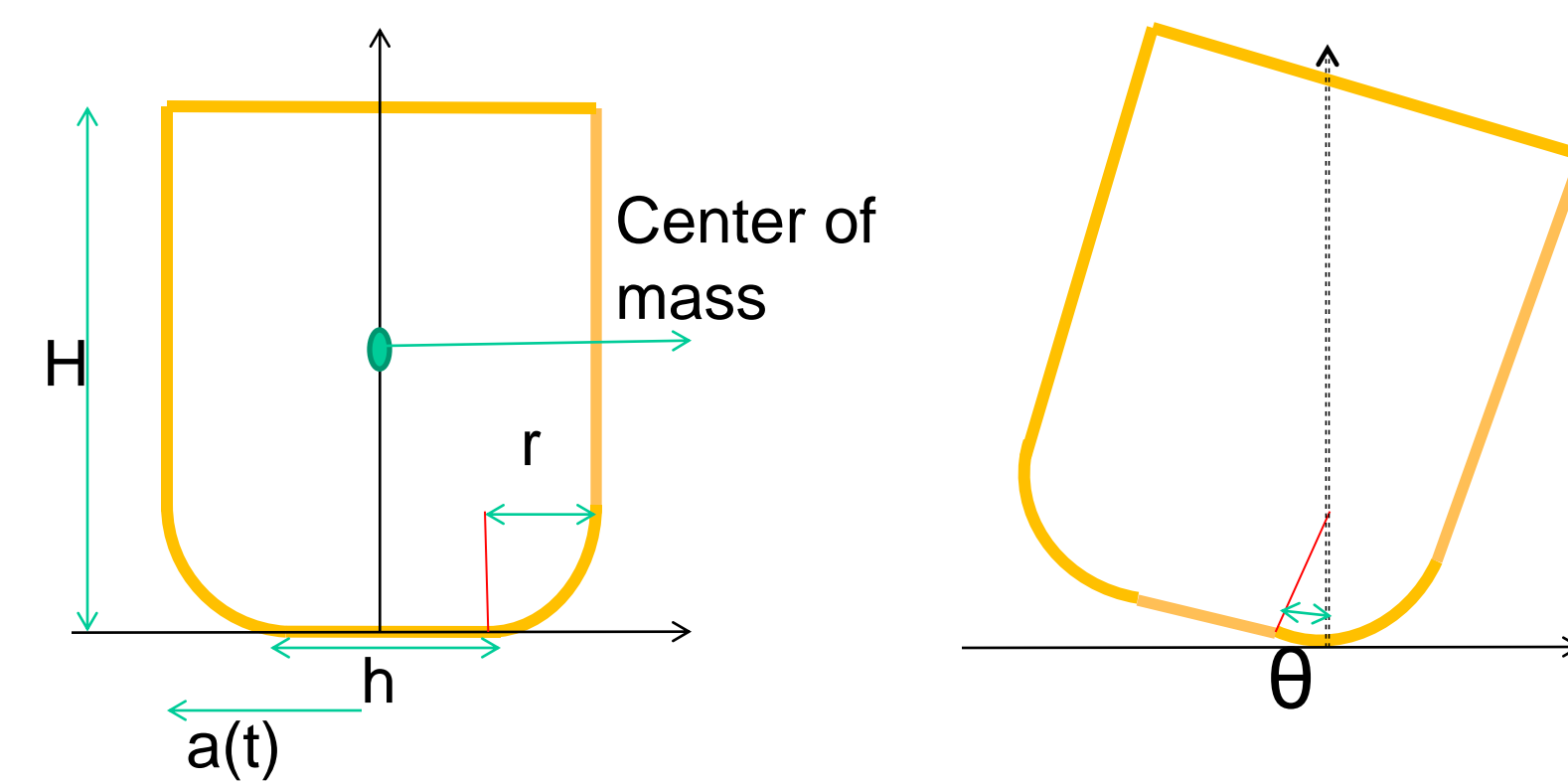


Figure 3: An idealized 2D block with a circular curvature

Here, h is the length of the flat part on the base and r is the radius of the circular curvature. H is the height of the block.

$$\text{Slenderness: } s = \frac{H}{2r+h}$$

$$\text{Basal Shape factor: } c = \frac{r}{h}$$

**Assumption:** No sliding and no bouncing.

This is a single degree of freedom system with  $\theta$  (angle rotated by a radius of the circular arc) as the degree of freedom. The equation of motion for this system can be obtained from the Lagrangian approach for a ground acceleration of  $a(t)$ .

$$\text{Equation of motion: } \ddot{\theta} = -\frac{\partial^2 L}{\partial \dot{\theta}^2} - \frac{g}{2I} \frac{dA_x}{d\theta} + \frac{a(t)}{I} \frac{dA_x}{d\theta} \quad \text{for } 0 < \theta < 90^\circ$$

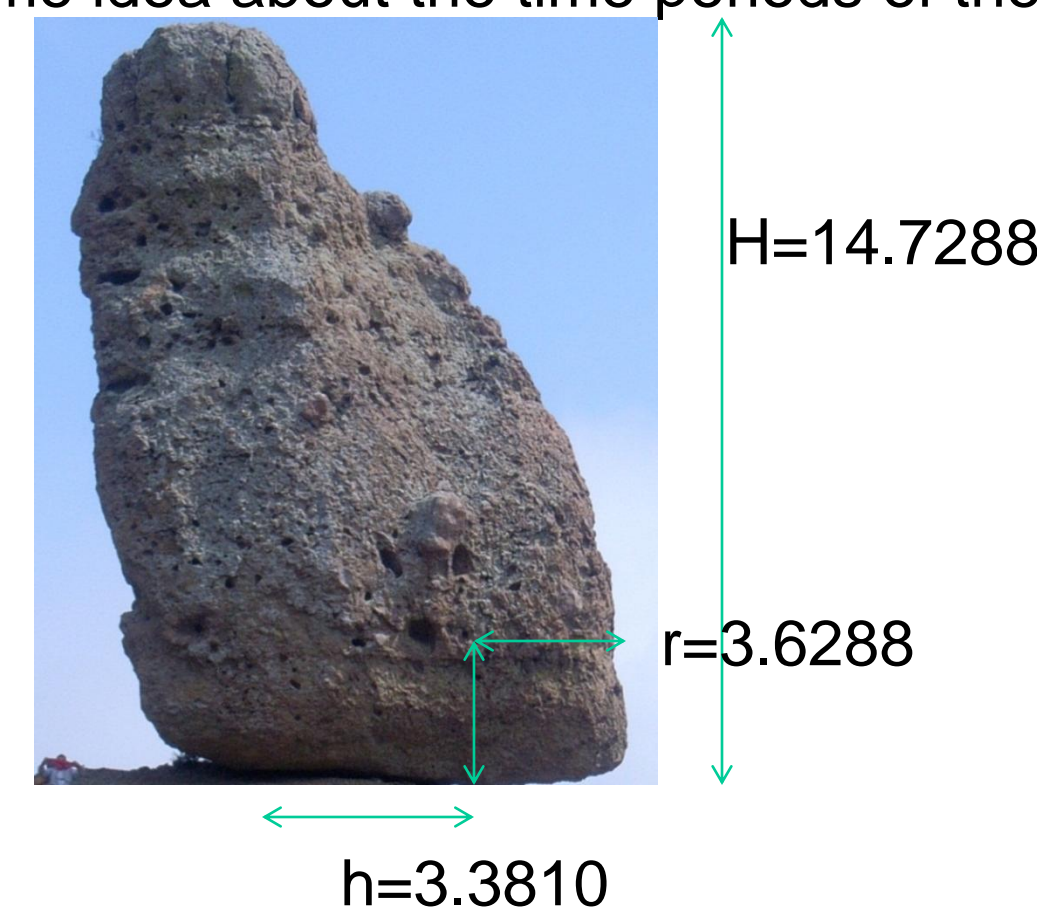
Where, I is the second moment of area about the current contact point,  $A_x$  is the first moment of area about the x axis and  $A_y$  is the first moment of area about the y axis passing through the current contact point. A similar equation exists for the other arc, i.e.  $-90^\circ < \theta < 0$ . Since we do not want the block to bounce when it moves from one arc to the other, we use the angular momentum conservation similar to what Housner derived.

Relation between angular velocities just before ( $\dot{\theta}_1$ ) and just after ( $\dot{\theta}_2$ ) transition from one circular arc to another:  $\dot{\theta}_2 = \left(1 - \frac{h^2}{2I}\right) \dot{\theta}_1$

This equation can be solved for any  $a(t)$  using Matlab.

### Natural Period of the block

For finding the natural period of the block, we will tilt the system and let it oscillate, i.e. apply initial angular displacement to the equation of motion. We will do this for a block with dimensions approximately same as that of the Echo Cliff rock in order to get some idea about the time periods of the rock.



Slenderness of the rock:  $s_r$

Basal Shape factor for the rock:  $c_r$

To see the effect of Slenderness and the basal shape separately, we will run the analyses keeping one parameter constant and varying the other.

**Time period changes with the angular displacement, basal shape factor and slenderness.**

Figure 4: Constant slenderness

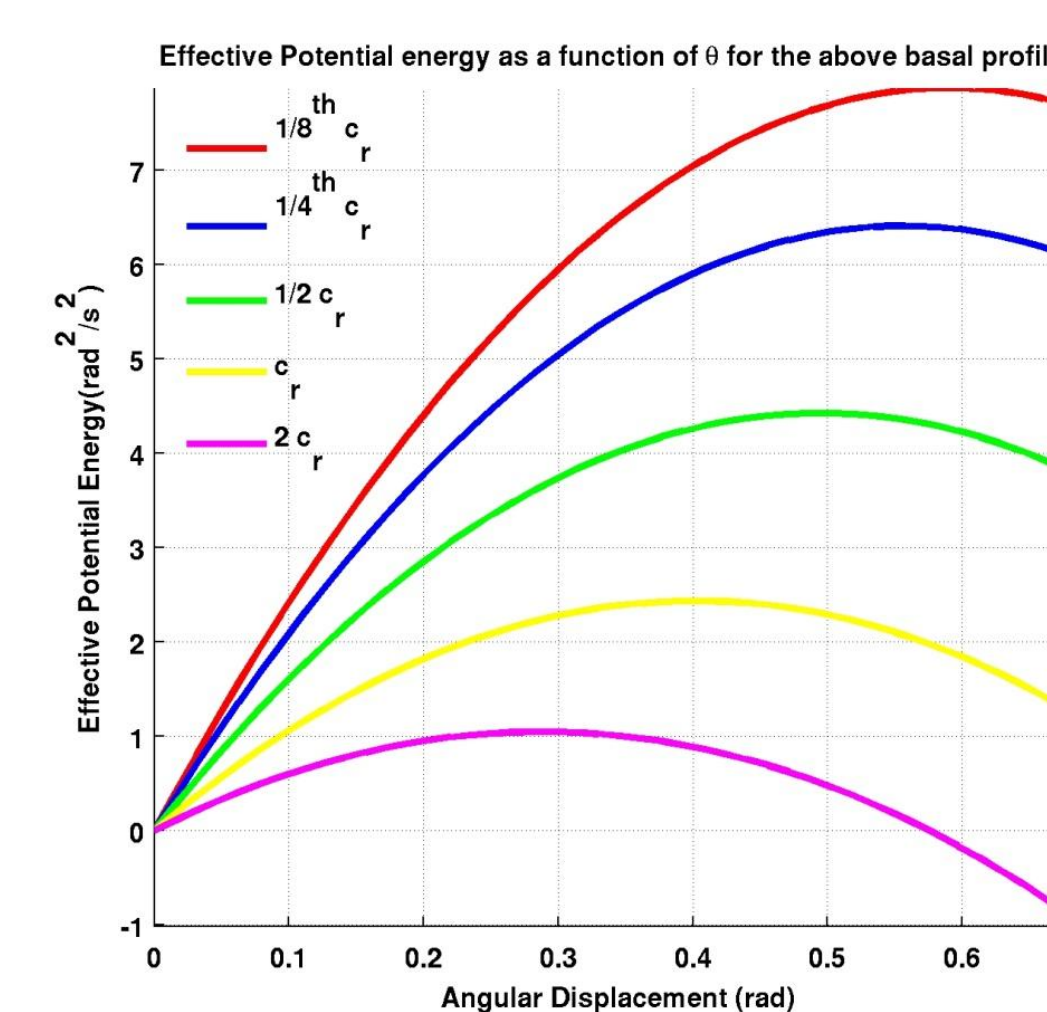
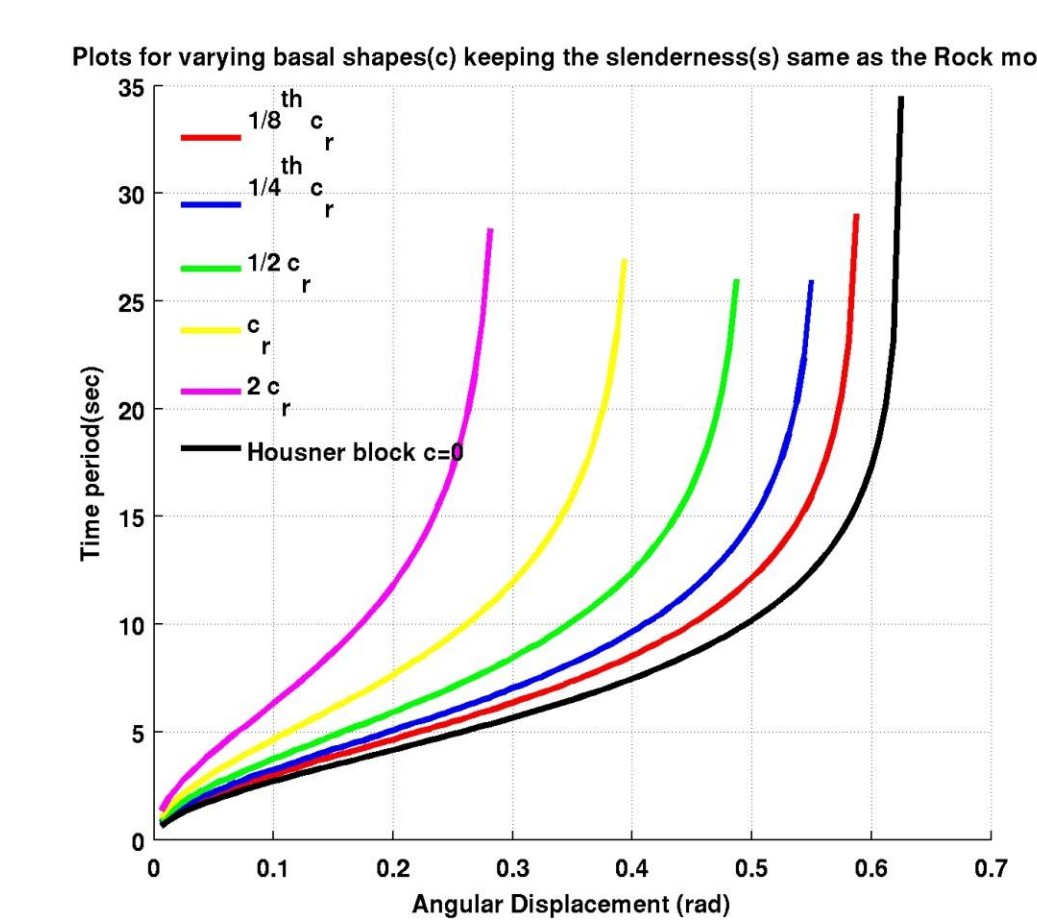
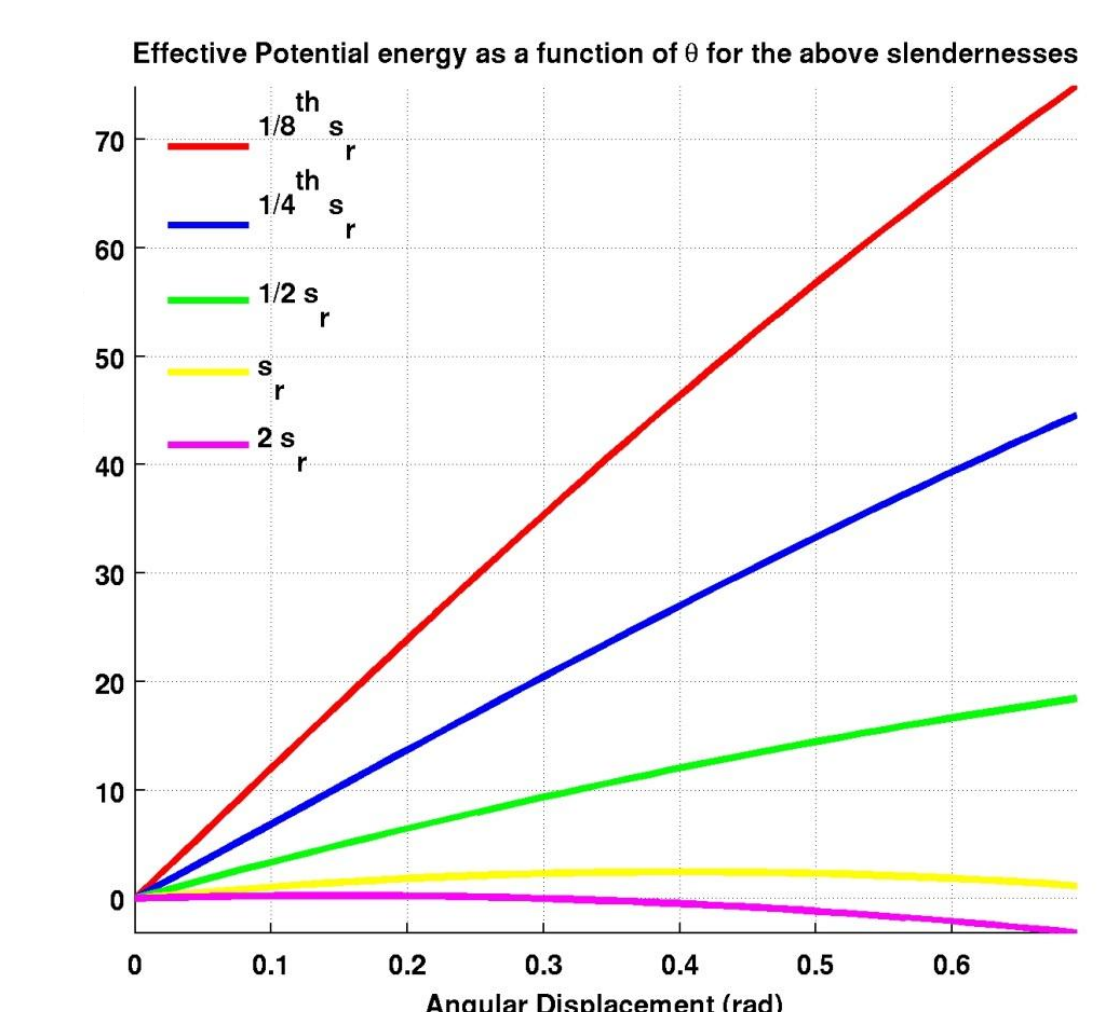
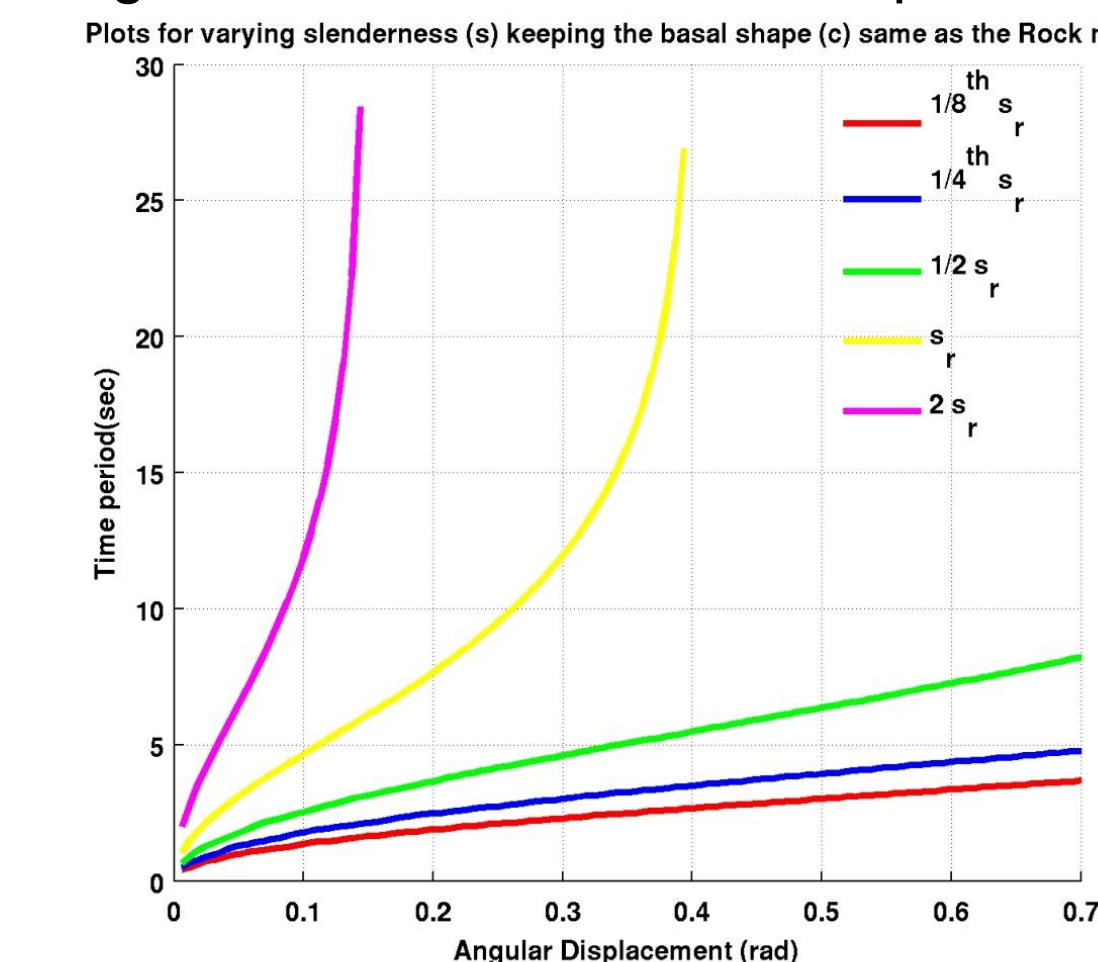


Figure 5: Constant Basal Shape

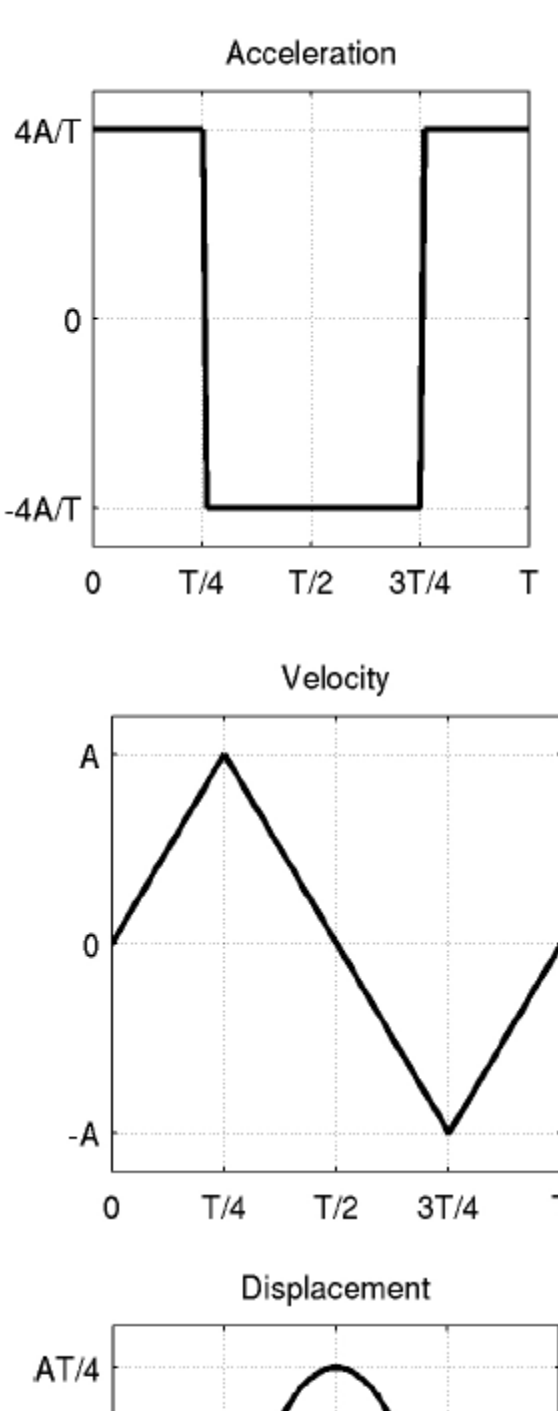


## References

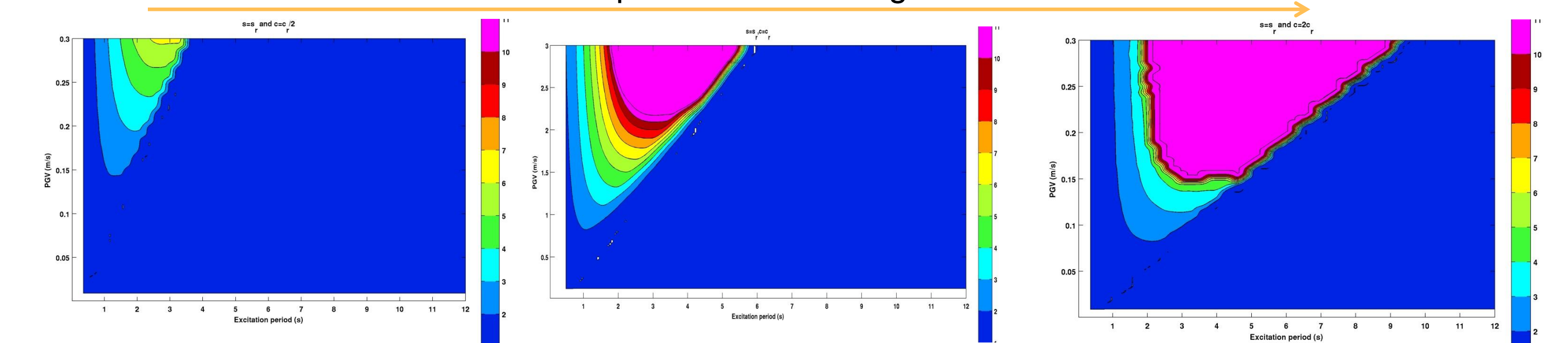
- (1) HOUSNER, G. W. The behavior of inverted pendulum structures during earthquakes. Bulletin of the Seismological Society of America 53, 2 (1963), 403-417.
- (2) BRUNE, J. N., ANOOSHEHPOOR, A., PURVANCE, M. D., AND BRUNE, R. J. Band of precariously balanced rocks between Elsinore and San Jacinto, California, fault zones: Constraints on ground motion for large earthquakes. Geology 34, 3 (2006), 137-140.

## Analysis for an idealized ground motion pulse

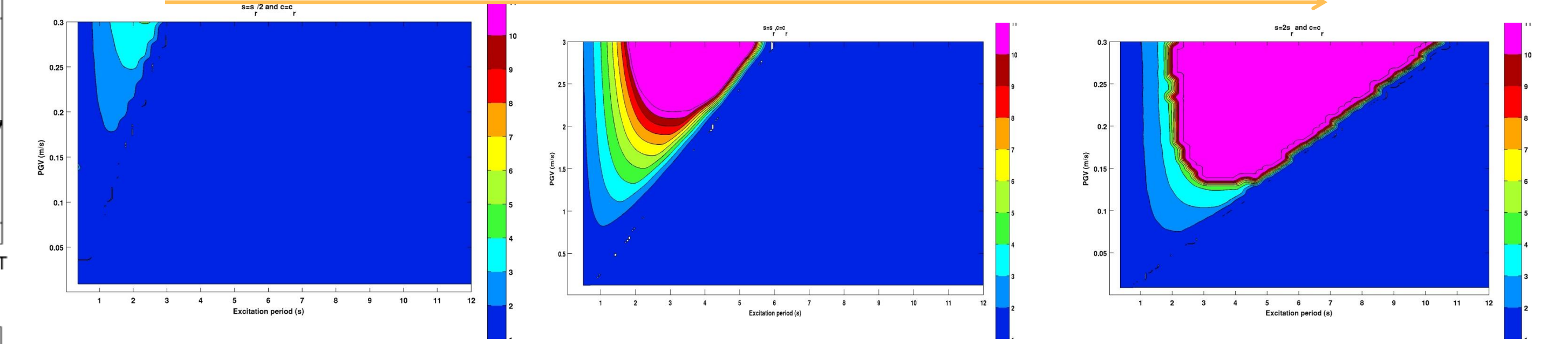
Ground motion Pulse



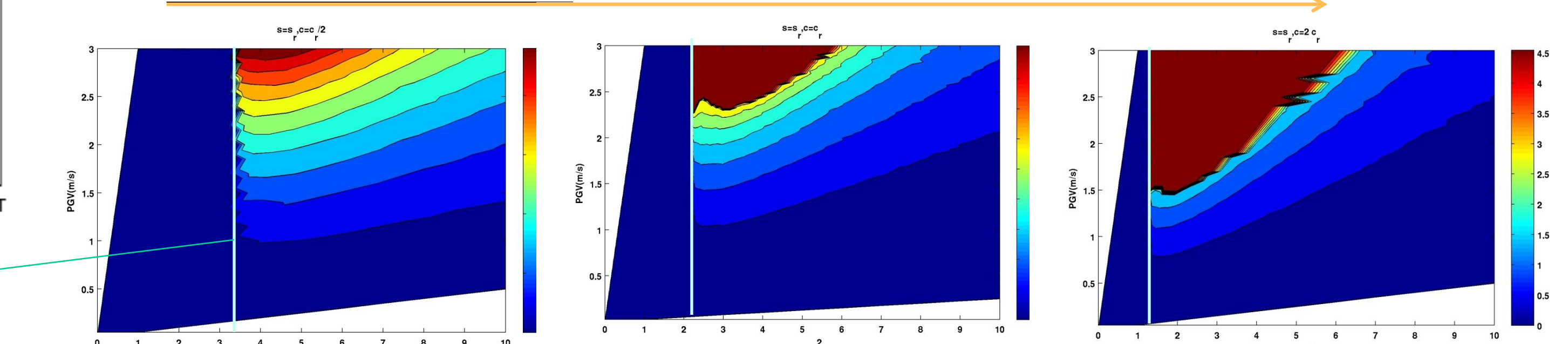
PGV vs T plots for Increasing Slenderness



PGV vs T plots for Increasing basal shape factor

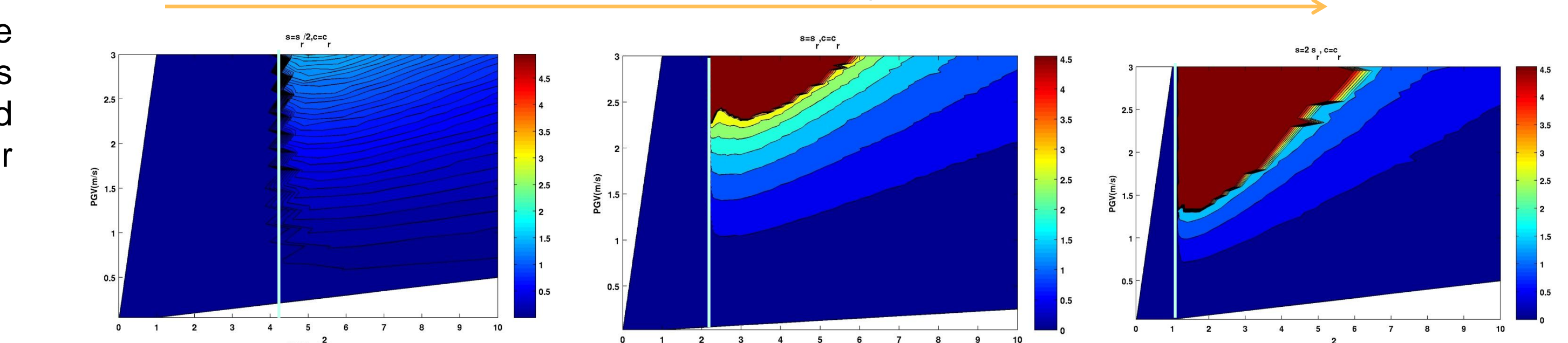


PGV vs PGA for Increasing Slenderness



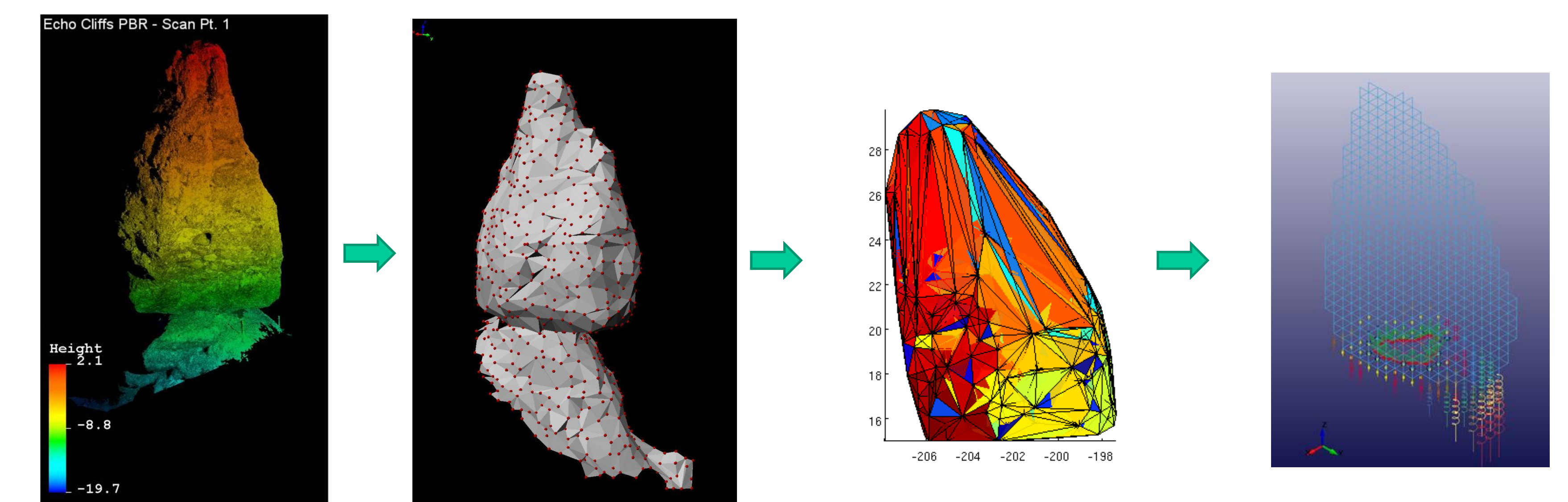
Quasi static toppling acceleration

PGV vs PGA for Increasing Basal Shape factor



The regions where the block topples increases as the slenderness and the basal shape factor increases.

## Preliminary model for the Echo cliff PBR



This is a preliminary model of the rock. We are still trying to get the accurate discretization by analyzing the FEM model for the 2D idealized rock. This mesh is created using Matlab and the analyses are carried out using FEM software LS-DYNA.

## Acknowledgments

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