Detecting Transient Signals in Geodetic Time Series Using Sparse Estimation

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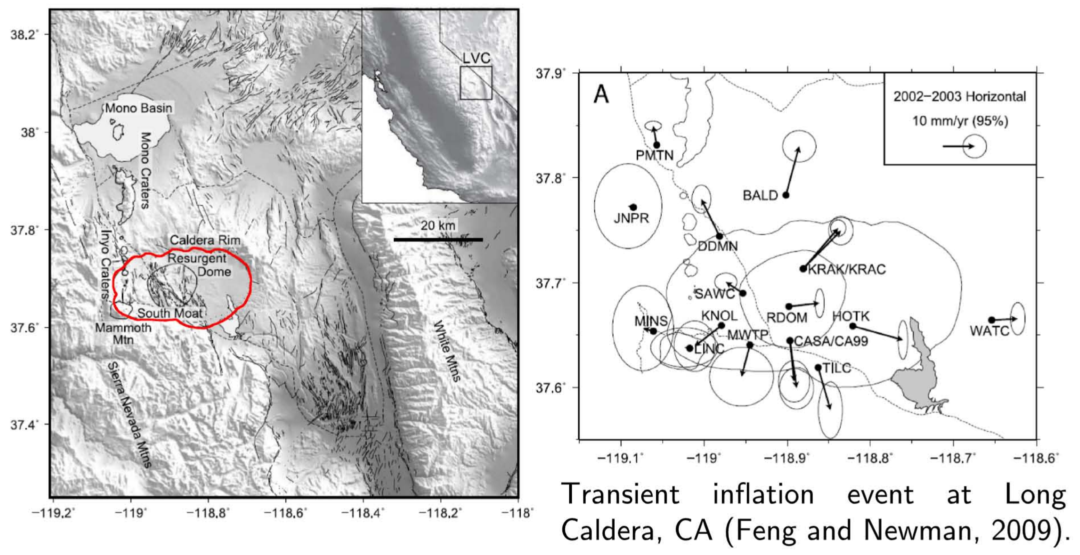
1. Introduction

Transients are a class of deformation signals that can be described as non-periodic, non-secular accumulation of strain in the crust. Examples include inflation/deflation over active geothermal areas, slow slip on plate interfaces, subsidence due to hydrologic activity, et al.



Transient inflation event at Long Valley Caldera, CA (Feng and Newman, 2009).

2. Challenges

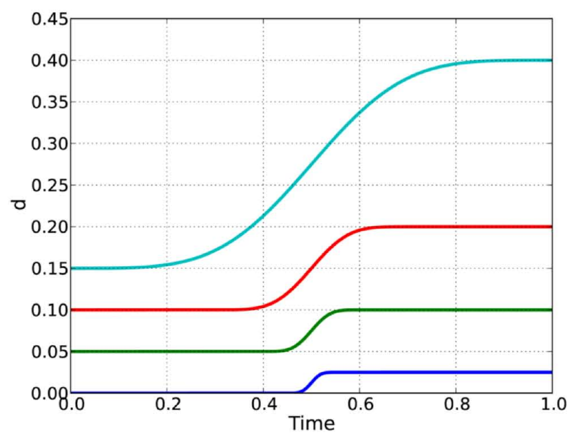
1) Detection difficulties due to noise and other modes of deformation.
2) Need to incorporate the **spatial coherency** of transients.
3) We often have no knowledge on the underlying physical mechanisms.
4) Method must be efficient for real-time detection capabilities.

3. Time parameterization

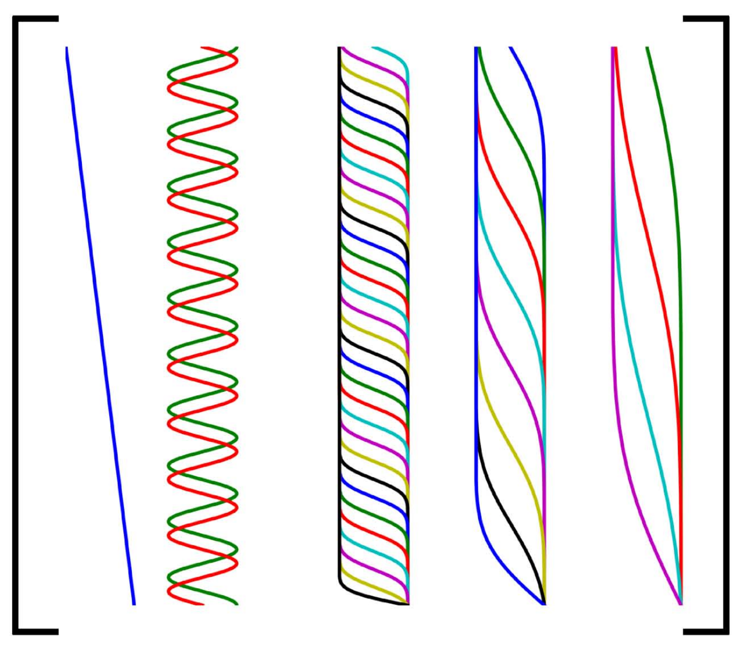
Without prior information about the physical mechanisms causing transient displacements, we cannot enforce rigid time parameterizations, i.e. exponential or logarithmic decay. We instead use flexible, generic mathematical functions that exhibit transient characteristics, mainly one-sided displacement.

- Use ***integral B-splines*** of different timescales to model transient signals of short and long durations.

- We set up a linear system of equations and solve for secular, seasonal, and transient signals.



$$\mathbf{Gm} = \mathbf{d}$$



$$\begin{pmatrix} m_1 \\ m_2 \\ m_3 \\ \vdots \\ m_P \end{pmatrix} = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ \vdots \\ d_N \end{pmatrix}$$

$N \times P$

4. Sparse Estimation

The transient detection problem is reduced to solving for the coefficients of the integral B-splines. Traditionally, we would seek to minimize the cost function:

$$\underset{\mathbf{m}}{\arg\min}\, J(\mathbf{m}) = \|\mathbf{d} - \mathbf{Gm}\|_2^2 + \lambda\|\mathbf{m}\|_2^2$$

However, the solution would not be sparse, i.e. we gain little insight into the timescales and amplitudes of the transient signals. We must instead use sparse estimation techniques which regularize using the L1-norm:

$$\underset{\mathbf{m}}{\arg\min}\, J(\mathbf{m}) = \|\mathbf{d} - \mathbf{Gm}\|_2^2 + \lambda\|\mathbf{m}\|_1, \qquad \|\mathbf{m}\|_1 = \sum_{i=1}^{P} |m_i|$$

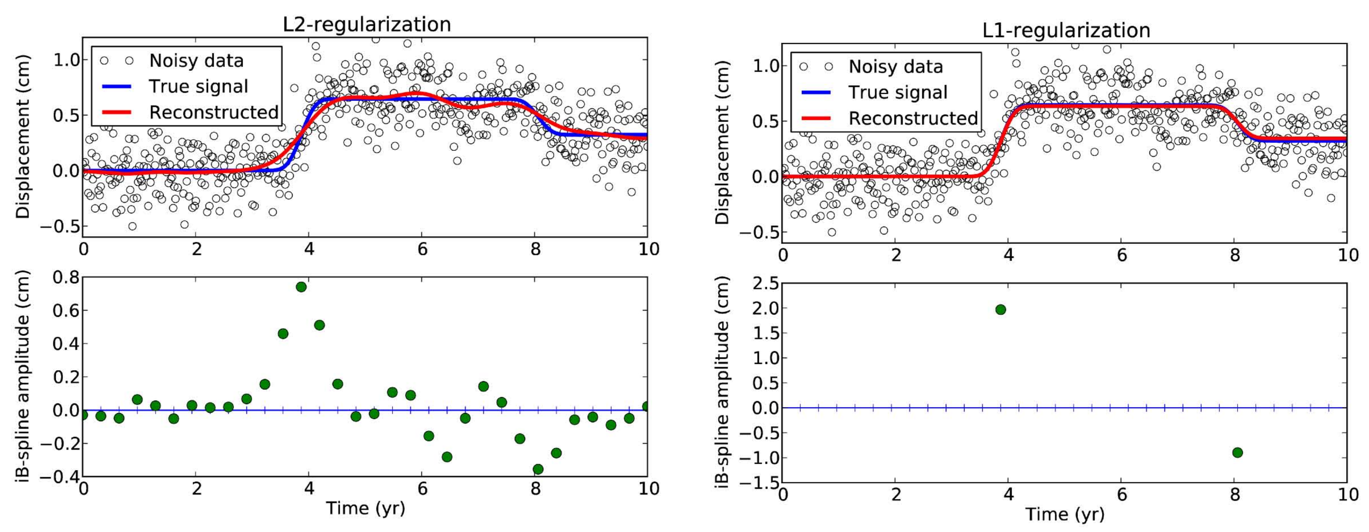


Figure 1. Synthetic transient recovery test. L2 case predicts non-zero values for all iB-spline coefficients and only roughly recovers true signal. L1 case nearly recovers the exact solution with only two non-zero coefficients.

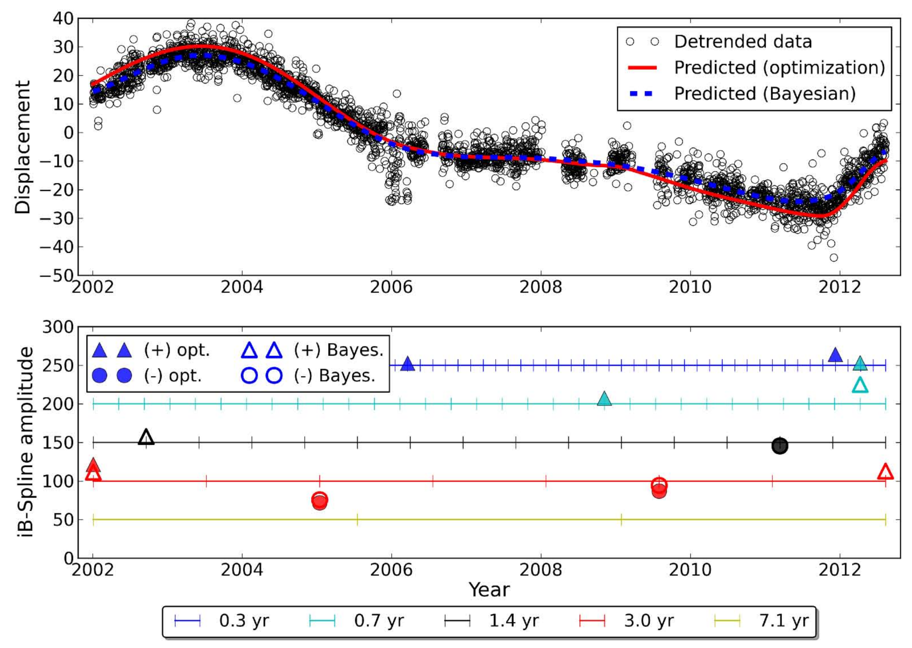
Bayesian / probabilistic framework: the maximum likelihood solution is proportional to the product of a normal distribution for the misfits and laplace prior distributions for the iB-spline coefficients -> sample the posterior!

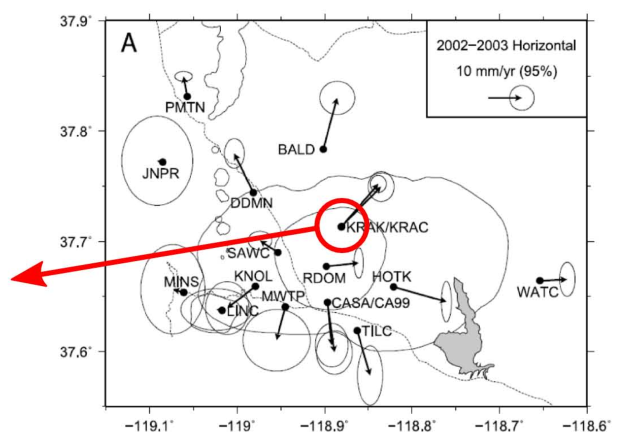
$$P(\mathbf{m}|\mathbf{d}) \propto P(\mathbf{d}|\mathbf{m})P(\mathbf{m})$$

$$\underset{\mathbf{m}}{\arg\max}\, e^{-J(\mathbf{m})} = e^{-(\mathbf{d}-\mathbf{Gm})^T \mathbf{C}_d^{-1}(\mathbf{d}-\mathbf{Gm})} \prod_i e^{-\lambda|m_i|}$$

$$\underset{\mathbf{m}}{\arg\max}\, e^{-J(\mathbf{m})} = \mathcal{N}(\mathbf{d}-\mathbf{Gm}, \mathbf{C}_d) \prod_i \mathcal{L}(m_i, \lambda)$$

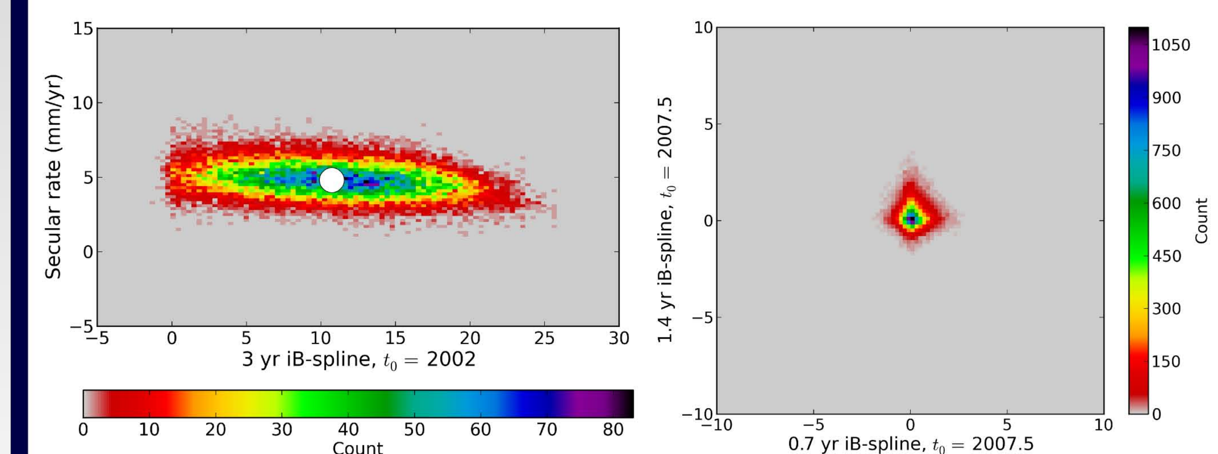
5. GPS Time Series: Long Valley Caldera





- Apply detection method on up-component of GPS station KRAC in the Long Valley Caldera, CA.

Figure 2. (top) Reconstructed transient time series; 2002-2003 inflation, 2004-2007 deflation, and rapid present-day inflation. (bottom) Selected non-zero iB-spline coefficients for 5 different temporal lengthscales.

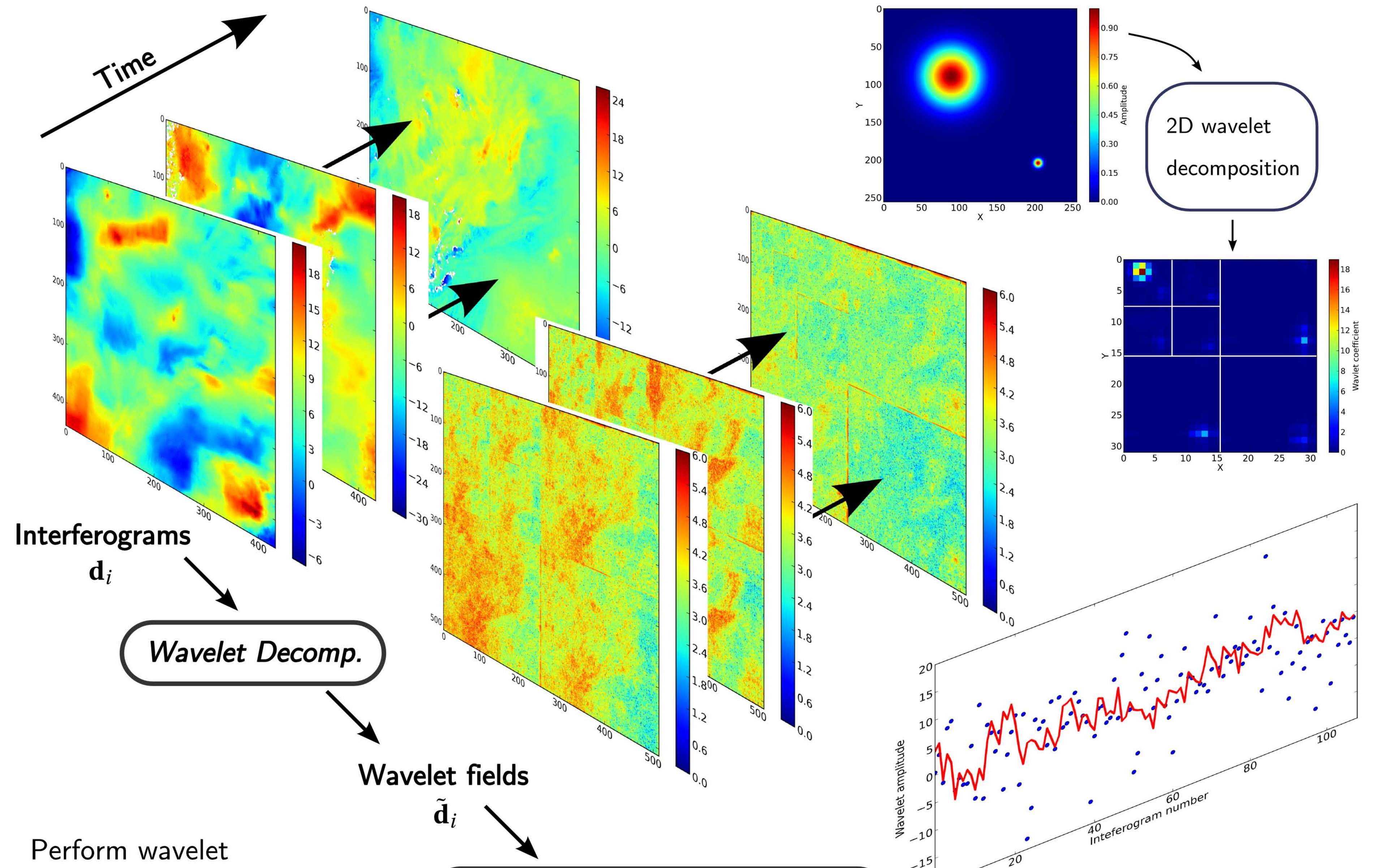


We can directly view coefficient covariances using the samples drawn from the Bayesian method.

Figure 3. 2-D histograms between (left) secular rate and 3-yr iB-spline corresponding to the 2002-2003 inflation, and (right) two iB-splines within quiescent period. The secular and 3-yr iB-spline slightly trade-off, i.e. some of the transient signal can also be accommodated by the secular rate. During quiescent periods, the laplace prior on the spline coefficients results in samples distributed tightly around the origin.

6. Wavelet Decompositions of InSAR Time Series

Two-dimensional wavelet decompositions of spatially continuous InSAR data allow us to characterize the location and effective scale of spatially coherent ground displacements.

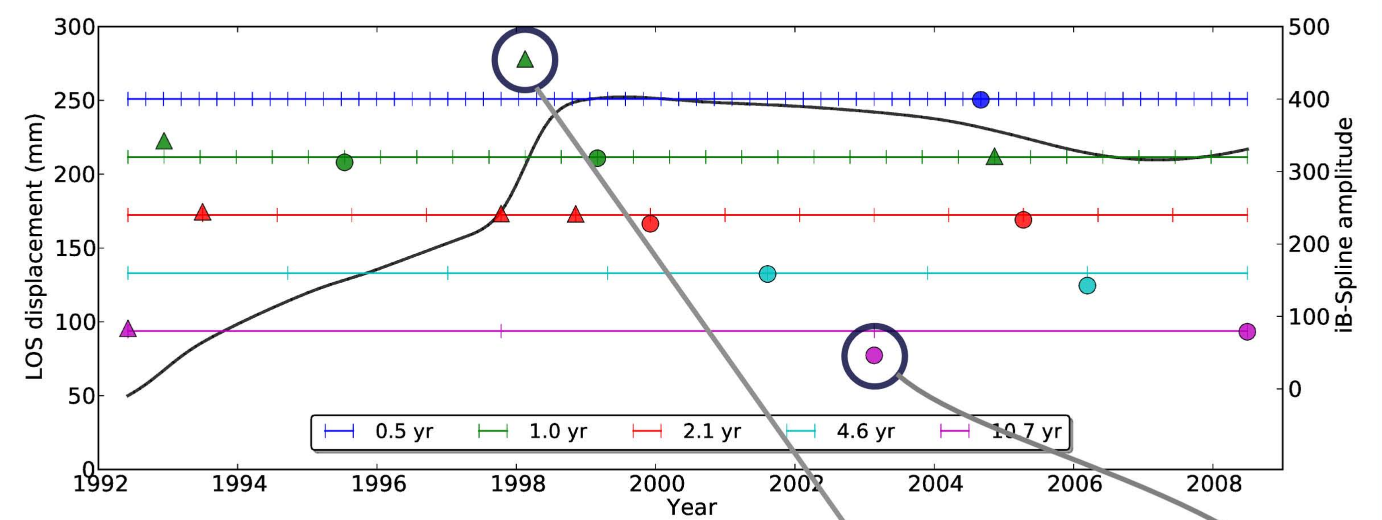


$$\underset{\tilde{\mathbf{m}}_i}{\arg\min}\, J(\tilde{\mathbf{m}}_i) = \|\tilde{\mathbf{d}}_i - \mathbf{G}\tilde{\mathbf{m}}_i\|_2^2 + \lambda\|\tilde{\mathbf{m}}_i\|_2^2$$

Perform wavelet decompositions on a stack of 263 ERS and Envisat interferograms over the Long Valley Caldera spanning the time period between 1992 and 2008. Solve for seasonal, secular, and transient signals for the time series of wavelet coefficients at each pixel. Use iB-splines of timescales of 0.5, 1.0, 2.3, 4.6, and 10.7 years.

Pixel-wise transient displacement history $\hat{\mathbf{d}}_i = \mathbf{G}\mathbf{m}_i$

Figure 4. Black curve displays the reconstructed LOS displacement for the pixel at the center of the resurgent dome of the caldera. The overlain markers display the amplitudes for the non-zero iB-spline coefficients at the same pixel.



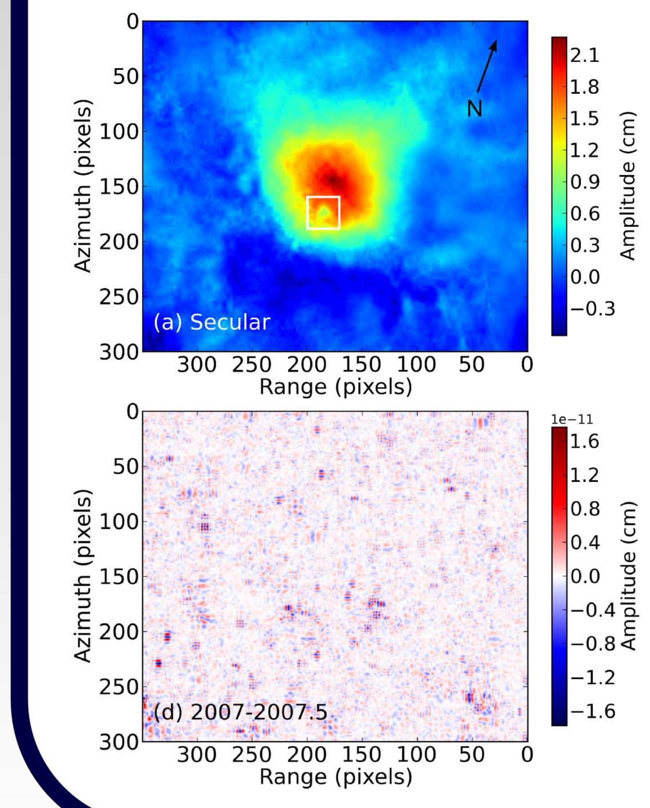


Figure 5. Spatial fields in radar coordinates for selected time series coefficients obtained after performing 2-D inverse wavelet transforms on the model wavelet coefficients $\tilde{\mathbf{m}}_i$. (a) Secular rate; note the subsidence of the geothermal field (white box) with respect to the caldera. (b) 1-yr iB-spline corresponding to 1997-1998 inflation event; distinct bulls-eye pattern showing spatially coherent ground uplift. (c) 10-yr iB-spline corresponding to subsidence after the 1997 inflation; very similar bulls-eye pattern suggesting deflation of the same source as the inflation event. (d) 0.5-yr iB-spline during 2007 quiescent period; generally very small amplitudes everywhere with no spatially coherent transient signals.