Inverse methods-based estimation of plate coupling in models governed by mantle flow

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Introduction

Plate motion is primarily controlled by buoyancy (slab pull) which occurs at convergent plate margins where oceanic plates undergo deformation near the seismogenic zone. Yielding within subducting plates, lateral variations in viscosity, and the strength of seismic coupling between plate margins likely have an important control on plate motion. We wish to infer the inter-plate coupling for different subduction zones along with the yield stress and strain rate exponent, and develop a method for inferring such quantities as a PDE-constrained optimization problem, where the cost functional is the misfit in plate velocities and is constrained by the nonlinear Stokes equation. We find that we can recover the plate boundary coupling along with either the yield stress or strain rate exponent in the upper mantle. However, it is harder to recover the plate coupling, yield stress and strain rate exponent without imposing a priori information. Moreover, we can recover the plate coupling in a continuously deforming region.

Mathematical Development

The inverse problem is given as follows where we seek to minimize the misfit in surface velocities:

$$\min_{\Gamma_i,\sigma_y,n} J(\Gamma_i,\sigma_y,n) = \int_{\Omega_{surface}} \|\boldsymbol{u}-\boldsymbol{u}_{data}\|_{\Gamma^{-1}_{noise}} dS_{surface} + \int \|m-m_0\|_{\Gamma^{-1}_{prior}}$$

Subject to the nonlinear Stokes equations and free slip boundary conditions.

$$\nabla\cdot\boldsymbol{u} = 0$$

$$\nabla\cdot\left[2\eta(u,\sigma_y,n,T)\dot{\epsilon}(\boldsymbol{u}) - p\mathbf{I}\right] = -Ra\,T\boldsymbol{e_y}$$

The PDE constraint is the nonlinear Stokes equations, with the nonlinearity arising from the viscosity dependence on the velocity. The viscosity $\boldsymbol{\eta(u,\sigma_y,n,T)}$ is

$$\eta(u,\sigma_y,n,T) = \begin{cases} \Gamma_i e^{[B(0.5-T)]}\epsilon_{II}^{\frac{1-n}{2n}}, & no\ yielding \\ \dfrac{[1-r]\sigma_y}{\epsilon_{II}^{1/2}} + r\,\Gamma_i e^{[B(0.5-T)]}\epsilon_{II}^{\frac{1-n}{2n}}, & yielding \end{cases}$$

With the adjoint equations:

$$\nabla\cdot\boldsymbol{v} = 0$$

$$\nabla\cdot\left[2\eta(\boldsymbol{u},\sigma_y,n,T)\left(I+\psi\frac{\dot{\epsilon}(\boldsymbol{u})\otimes\dot{\epsilon}(\boldsymbol{u})}{Tra(\dot{\epsilon}(\boldsymbol{u})^2)}\right)\dot{\epsilon}(\boldsymbol{v}) - q\mathbf{I}\right] = 0$$

With boundary Conditions:

$$\boldsymbol{v}\cdot\boldsymbol{n} = 0$$

$$\mathrm{T}\left[2\eta(\boldsymbol{u},\sigma_y,n,T)\left(I+\psi\frac{\dot{\epsilon}(\boldsymbol{u})\otimes\dot{\epsilon}(\boldsymbol{u})}{Tra(\dot{\epsilon}(\boldsymbol{u})^2)}\right)\dot{\epsilon}(\boldsymbol{v}) - q\mathbf{I}\right]\cdot\boldsymbol{n} = -(\boldsymbol{u}-\boldsymbol{u}_{data})$$

Observational data is usually contaminated with noise. Moreover, the model of the physical system has uncertainties. This can be represented as:

$$u = u_{data} + \phi$$

Where we assume that the noise is Gaussian and i.i.d (independent and identically randomly distributed). The covariance matrix from noise is:

$$\phi = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-x^*)^2/2\sigma^2}$$

With the posterior distribution being:

$$\pi_{post} \approx \pi_{like}(u_{obs}|\boldsymbol{m})\pi_{prior}(\boldsymbol{m})$$

The solution to the minimization problem gives us the maximum a posteriori point (MAP), which is the mean of the Gaussian approximation of the posterior distribution. Since plate velocities are give by kinematic models (MORVEL, NUVEL-1A, etc.) we need to use a similar way to impose surface velocities. We ascribe a constant to regions of a plate for the surface velocity given by the RMS of the velocity in that region.

Plate Boundary Coupling

In order to infer the rheological parameters, we need to solve the second order system given below. Rheological parameters such as the strain rate exponent, yield stress and coupling factor play an integral role in how couple plate boundaries are.

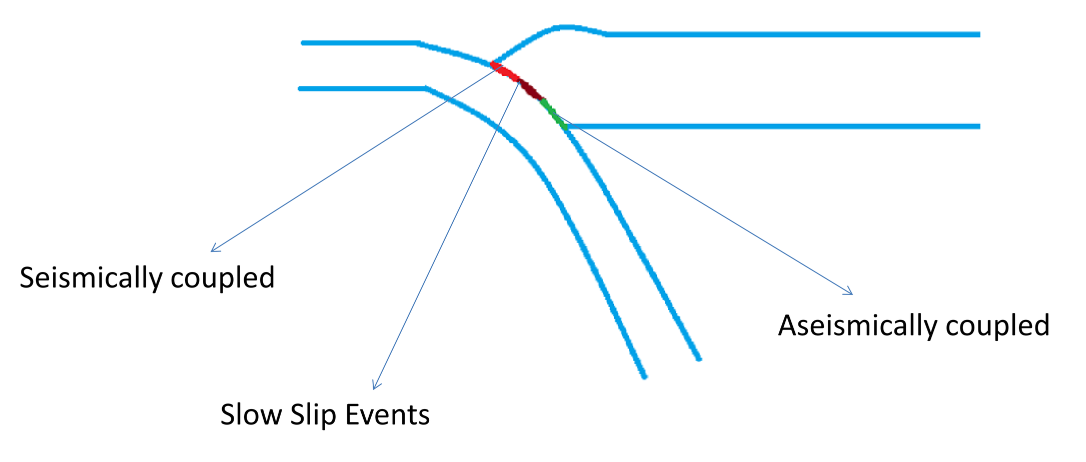

Figure 1: Subduction Zone with seismogenic interface

We compute the shear stress from the rheological relationship:

$$\sigma_{shear/normal} = \left(\int |2\eta(u,n,\sigma_y,T)\epsilon_{II}|^2 d\Omega\right)^{\frac{1}{2}}$$

Case Studies

For all case studies , we consider a subduction zone system with three subducting oceanic plates. The left most plate penetrates into the lower mantle, while the middle plate does not reach the lower mantle, and the right-most plate just touches the upper-lower mantle interface. Additionally, there is back-arc spreading to the left of the middle subducting plate.

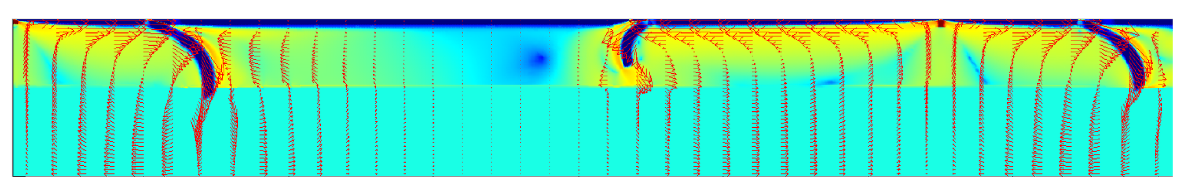

Figure 2 : Effective Viscosity with velocity Field

Case I: Inferring Plate Coupling with Back Arc Spreading

Here, with known strain rate exponent and yield stress, we infer the plate coupling for each subducting plate. We are able to recover the true shear stresses in each plate boundary with synthetic data imposed on each plate while constraining plate motion to observed data.

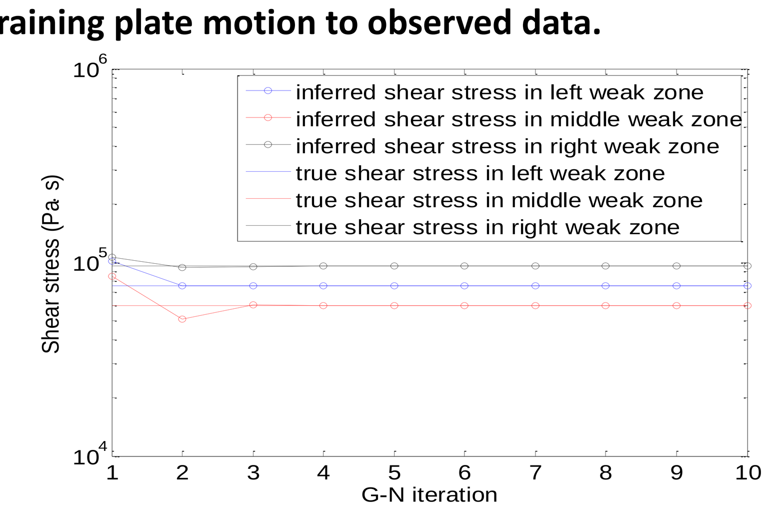

Figure 3: Recovered Shear Stress

Case II: Inferring Plate Coupling

For this case we infer the plate coupling and the yield stress. We match the surface velocity and recover the plate coupling and yield stress.

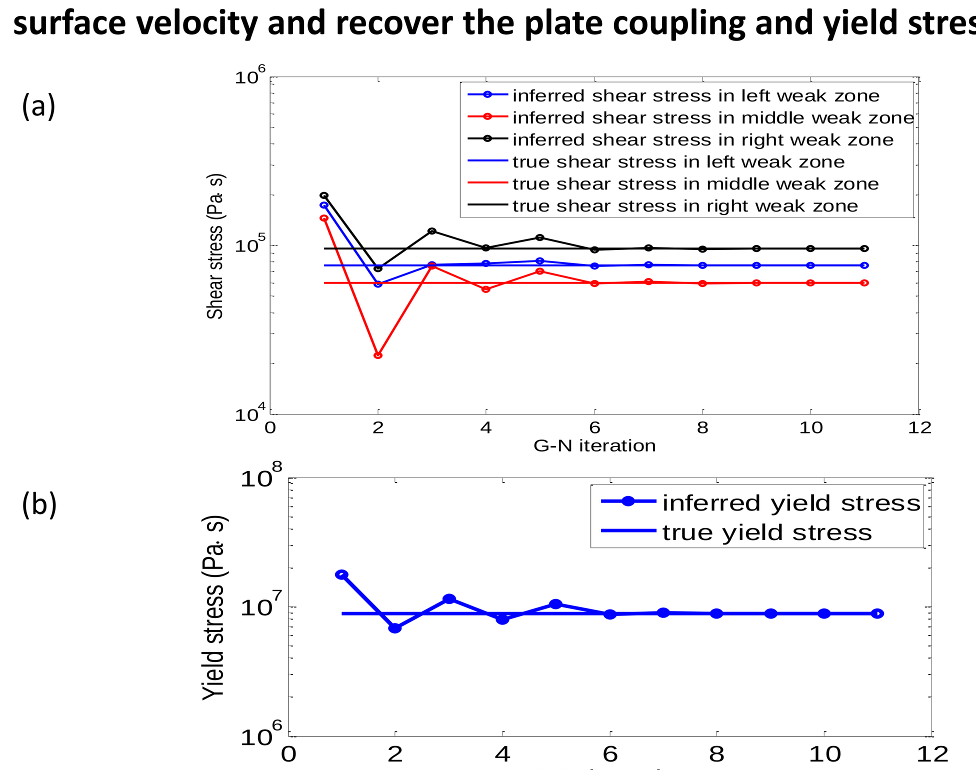

Figure 4:(a) Recovered Shear Stress (b) Recovered Yield Stress

Case II: Plate Motion sensitivity to plate coupling

As one makes plate boundaries weaker, there reaches a point where the plate motion becomes insensitive to the plate boundary coupling. As shown below, after a value of approximately $\Gamma_i = 10^{-12}$, the RMS of the surface velocity reaches a steady state. Moreover, as plates become more coupled, such that the system of plates move together as one plate, one reaches another steady state where the RMS surface velocity reaches a minimum.

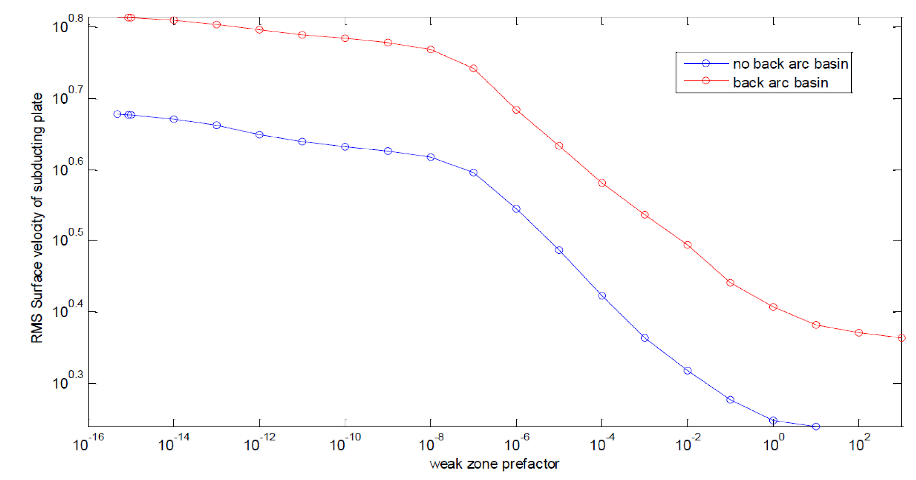

Figure 5: Plate Motion Sensitivity Plot

Trade-offs in Parameter Estimation

When plate coupling, yield stress, and strain rate exponent are unknowns, we are not able to recover the true rheological parameters.

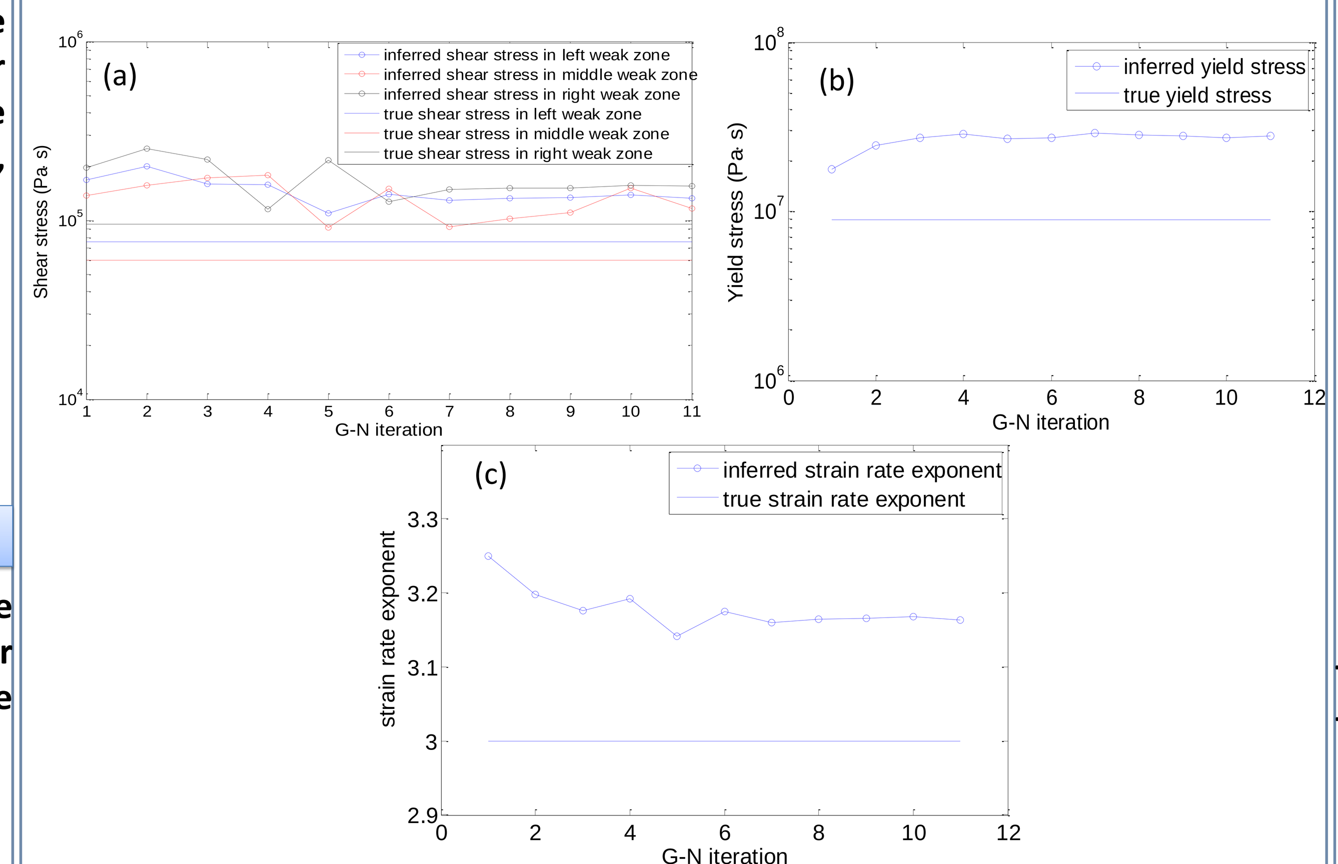

Figure 6:(a) Recovered Shear Stress (b) Recovered Yield Stress (c) Recovered strain rate exponent

The trade-off in this poorly constrained case shows that an increase in the strain rate exponent needs an increase in yield stress to slow plate.

Case II: Bounding the Coupling

To minimize the extreme tradeoffs in rheological parameters, we bound the plate-coupling away from the insensitive region by prescribing the weak factor in each plate boundary to be greater than 10^{-10}. Doing so, we are able to recover the correct order of plate boundary strength.

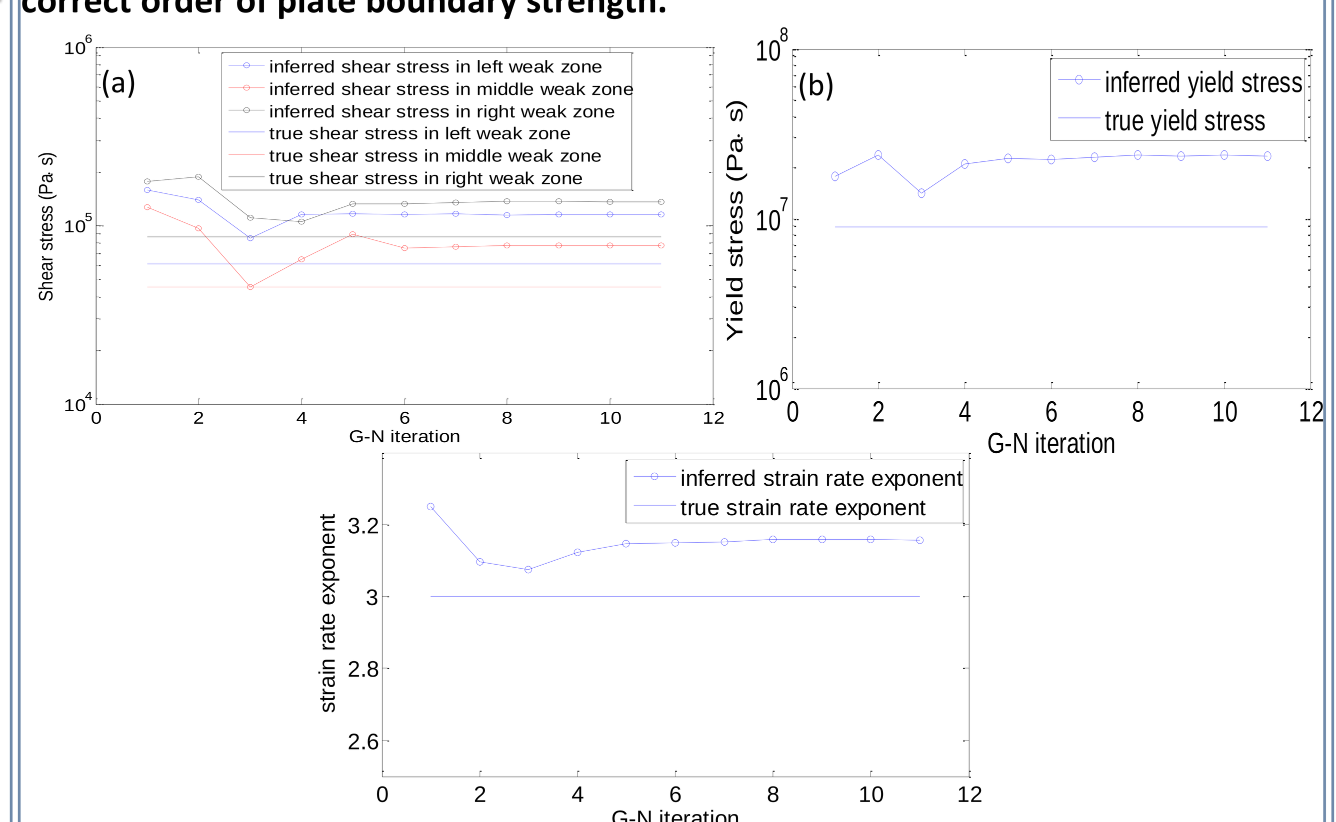

Figure 7:(a) Recovered Shear Stress (b) Recovered Yield Stress (c) Recovered strain rate exponent

Recovery of Plate Coupling in a continuously deforming region

There are regions of the earth that undergo continuous deformation which can be seen from GPS velocity maps and strain rate fields. We explore a similar case where one of the plate boundaries is continuously deforming. Moreover, we assume that the velocity data for the deforming boundary is contaminated with noise:

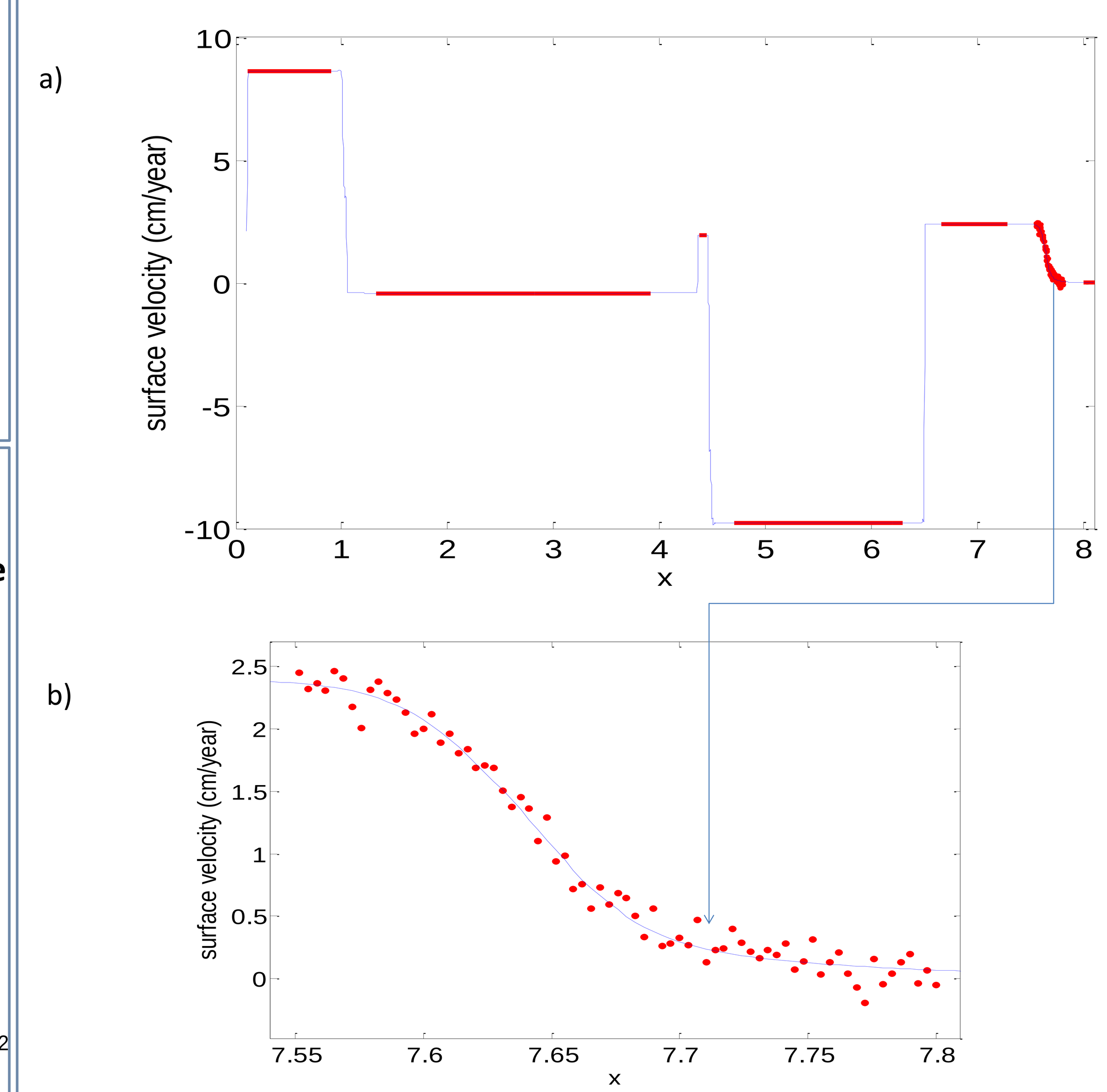

Figure 8 : Surface Velocity: Red lines denote observed data (b) Zoom in of velocity data of deforming boundary (red dots denote observed velocity contaminated with 5 percent noise).

The weakening factor and its relative error from the inversion are shown in the figures below.

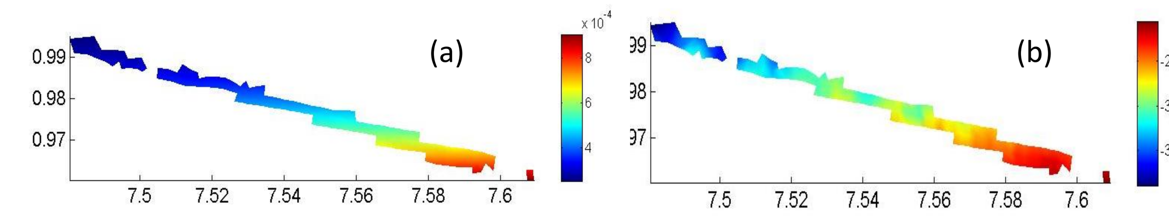

Figure 9 : Weakening factor (log_{10}) of the right most weak zone (continuously deforming) (b) relative error (log_{10}) of inferred weakening factor of continuously deforming region.

We are able to recover the shear stress in all plate boundaries as shown below.

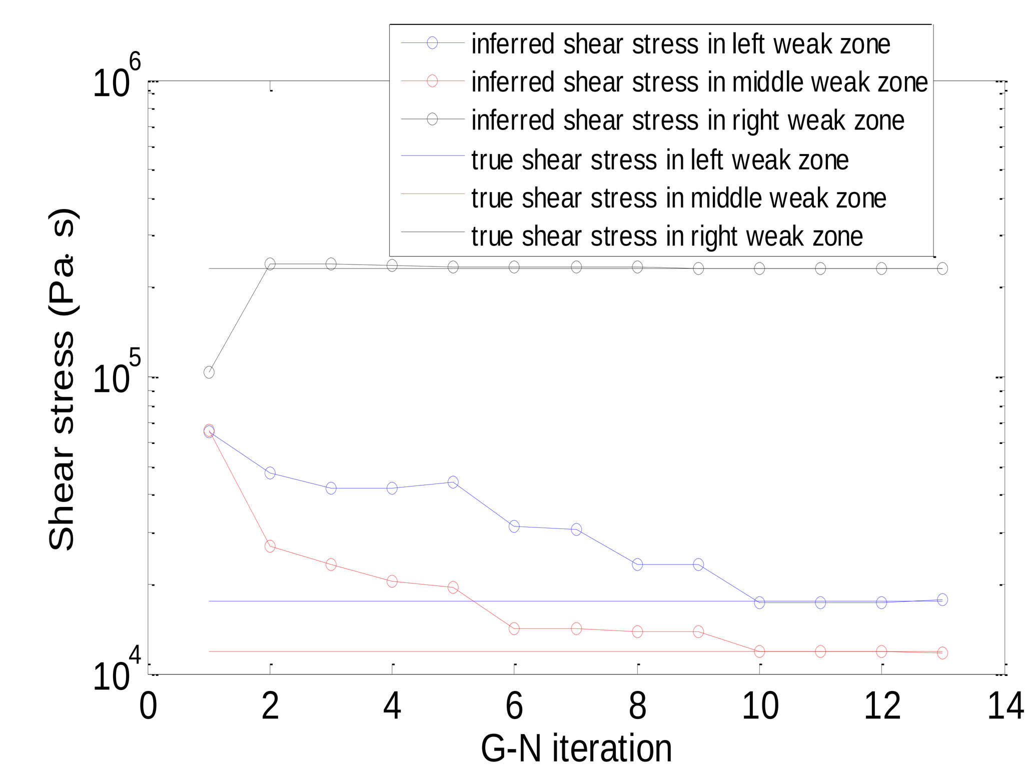

Figure 10 : Inversion for shear stresses in each plate boundary.