

Supplemental Information: Equations used in Figure 4

The relations used to compute the curves in Figure 4 were first published by Whipple and Meade¹. That paper did not, however, provide full details on the relations for the hypothetical fixed-width mountain range used in this review as a foil for comparison to predictions for critical-taper systems. All relations used for the fixed-width case in Figure 4 are reported below for the convenience of the interested reader.

All calculations use the orogen-scale erosion rule² given by:

$$E = C \left(\frac{1}{2} W \right)^a (\tan \alpha)^b, \quad (1)$$

where E is erosion rate (positive downward), C is the coefficient of erosional efficiency (a function of climate, rock properties, and sediment load), $W/2$ is the range half-width, and $\tan \alpha$ is the regional topographic gradient (or topographic taper).

For simplicity, for the fixed-width system computations, I assume $a = b = 1$. Noting that range relief (R) for a symmetrical range is by definition the product of regional topographic gradient and range half-width ($R = 1/2 W (\tan \alpha)$), in this case the orogen-scale erosion rule reduces to a simple linear function of relief³:

$$E = CR \quad (2)$$

Following England and Molnar⁴, I define surface uplift (U_s) as the difference between near-surface rock uplift (U , measured relative to fixed baselevel) and erosion rate:

$$U_s \equiv U - E \quad (3)$$

Near-surface rock uplift rate is specified because the vertical component of rock velocity often varies with depth – equation (3) is only valid at or near the surface.

The surface uplift rate is by definition equal to the rate of change of relief (dR/dt). For an isostatically compensated range, the rate of surface uplift is a function of the change in crustal thickness, which is determined by the difference between the tectonic rate of crustal thickening (F_A/W) and the rate of erosion ($E = CR$), and the densities of crust (ρ_c) and mantle (ρ_m) (Airy isostasy):

$$\frac{dR}{dt} \equiv U_s = \left(\frac{\rho_m - \rho_c}{\rho_m} \right) \left(\frac{F_A}{W} - CR \right) \quad (4)$$

This well-known differential equation can be solved directly:

$$R(t) = R_f + (R_i - R_f) e^{-C \left(\frac{\rho_m - \rho_c}{\rho_m} \right) t} \quad (5)$$

where R_i is the initial steady state, R_f is the final steady state and equation 5 describes the evolution of range relief following a change in climate from an initial state (C_i) to a final state (C) at time $t = 0$. Initial and final steady-state relief is found by setting $dR/dt = 0$ in equation 4:

$$R_i = F_A / (WC_i); \quad R_f = F_A / (WC) \quad (6)$$

The exponential relation in equation (4) has a characteristic response time (equal to the e-folding time) given by:

$$\tau = 1 / C \left(\frac{\rho_m - \rho_c}{\rho_m} \right) \quad (7)$$

All variables in Figure 4 are plotted against non-dimensional time ($T^* = t/\tau$). A relation for erosion rate is found by combining equation (2) and (5):

$$E(t) = CR_f + C(R_i - R_f) e^{-C \left(\frac{\rho_m - \rho_c}{\rho_m} \right) t} \quad (8)$$

Total erosional efflux is of course the product of erosion rate and range width (W). Rock uplift rate is found by substituting (8) into (3) and solving for rock uplift rate using (4):

$$U(t) = \frac{\rho_c}{\rho_m} C(R_i - R_f) e^{-C \left(\frac{\rho_m - \rho_c}{\rho_m} \right) t} + \frac{F_A}{W}$$

All computations involving the critical-taper wedge case were done using relations in Whipple and Meade¹ (their equations 7, 14, 15) and assuming $p = 1$ in their equation (6) for simplicity.

References Cited

- ¹ Whipple, K. X. & Meade, B. J. Orogen response to changes in climatic and tectonic forcing. *Earth and Planetary Science Letters* **243**, 218-228 (2006).
- ² Whipple, K. & Meade, B. J. Controls on the strength of coupling among climate, erosion, and deformation in two-sided, frictional orogenic wedges at steady state. *J. Geophys. Res.* **109**, doi:10.1029/2003JF000019 (2004).
- ³ Ahnert, F. Functional relationships between denudation, relief, and uplift in large mid-latitude drainage basins. *Am. J. Sci.* **268**, 243-263 (1970).
- ⁴ England, P. & Molnar, P. Surface uplift, uplift of rocks, and exhumation of rocks. *Geology* **18**, 1173-1177 (1990).